

# Strong decay patterns of the hidden-charm tetraquarks

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With the spin rearrangement, we have performed a comprehensive investigation of the decay patterns of the S-wave tetraquarks and P-wave tetraquarks where the P-wave excitation exists either between the diquark and anti-diquark pair or inside the diquark. Especially, we compare the decay patterns of  $Y(4260)$  with different inner structures such as the conventional charmonium, the molecule, the P-wave tetraquark and the hybrid charmonium. We notice the  $J/\psi\pi\pi$  mode is suppressed in the heavy quark symmetry limit if  $Y(4260)$  is a molecular state. Moreover the hybrid charmonium and hidden-charm tetraquark have very similar decay patterns. Both of them decay into the  $J/\psi\pi\pi$  and open charm modes easily. We also discuss the decay patterns of  $X(3872)$ ,  $Y(4360)$ , and several charged states such as  $Z_c(4020)$ . The  $h_c\pi^\pm$  decay mode disfavors the tetraquark assumption of  $Z_c(4020)$ .

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## I. INTRODUCTION

Since  $X(3872)$  was observed by Belle Collaboration in the  $J/\psi\pi^+\pi^-$  invariant mass spectrum of  $B \rightarrow KJ/\psi\pi^+\pi^-$ , many charmonium-like and bottomonium-like states have been reported by the Belle, CMS, LHCb, CDF, D0, CLEO-c and BESIII Collaborations [1–8]. Some states pose a great challenge to the conventional quark model. Especially, the charged charmonium-like and bottomonium-like states such as  $Z_c(3900)$ ,  $Z_c(4025)$  and  $Z_1(4475)$  do not fit into the conventional charmonium spectroscopy [9–16, 18, 19, 60]. Up to now, these charged  $Z_c$  and  $Z_b$  states seem to be the best candidates of the four-quark states. Very recently, two hidden-charm pentaquarks  $P_c(4380)$  and  $P_c(4450)$  were reported by LHCb in the  $J/\psi p$  invariant mass spectrum of the  $\Lambda \rightarrow J/\psi p K$  process [20], which enriched our knowledge of the hidden-charm multiquark systems [21–23].

Many theoretical speculations have been proposed to interpret these XYZ states, such as the kinematics artifacts, conventional charmonium, hybrid charmonium, di-meson molecules and tetraquarks [3, 24–39]. Since some states are very close to the open charm or open bottom threshold, the molecule assumption becomes quite popular among the theoretical proposals. For instance, many authors suggest  $X(3872)$  as a possible candidate of the  $D\bar{D}^*$  molecules [38, 40–50].  $Y(3940)$ ,  $Y(4140)$  and  $Y(4274)$  were proposed as the  $D^*\bar{D}^*$ ,  $D_s^*\bar{D}_s^*$  and  $D_s\bar{D}_{s0}(2317)$  molecular states in Ref. [53–61, 70, 71]. The two charged bottomonium-like states  $Z_b(10610)$  and  $Z_b(10650)$  were suggested as the loosely bound S-wave  $B\bar{B}^*$  and  $B^*\bar{B}^*$  molecules [62, 63]. There were many discussions of the possible molecular assignment of the XYZ states in literature [64–71].

The hidden-charm molecular states generally lie close to the open-charm threshold. They were first observed in the hidden-charm final states such as  $J/\psi\pi$ . However their open-charm decay widths are much larger as in the case of  $Z_c(3900/4020)$ .

Besides the molecular scheme, the tightly bound tetraquark states are also very interesting. They are expected to be very broad with a width around several hundred MeV. Moreover, they tend to decay into the hidden-charm modes easily. With a larger phase space, the hidden-charm modes should be one of their main decay modes. For example, the charged state  $Z_c(4200)$  decays into  $J/\psi\pi$ . Its decay width is more than three hundreds MeV [72]. The decay pattern of  $Z_c(4200)$  as a tetraquark candidate was investigated with the QCD sum rule formalism recently [74]. The spectrum of the hidden-charm tetraquarks were calculated systematically in Refs. [75–81]. There were other discussions of the XYZ states as candidates of the tetraquarks [82, 83].

$Y(4260)$  was observed in the  $J/\psi\pi\pi$  mode using the initial state radiation technique. Many speculations of its inner structure have been proposed in the past decade [33, 84–99]. This vector charmonium-like state is quite broad. However, its main decay modes have not been observed up to now. The non-observation of  $Y(4260)$  in many decay modes is very puzzling.

Especially, there is no evidence of the open charm decays for  $Y(4260)$ , which poses a great challenge to the molecule assumption. With the spin rearrangement scheme in heavy quark limit, Ma et al. investigated the decay patterns of the hidden-charm molecular states and their typical decay ratios [67, 68]. The authors found that the  $1^{--}$  molecular states composed of  $D_0\bar{D}^*$ ,  $D_1'\bar{D}$ ,

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$D_1\bar{D}$ ,  $D_1'\bar{D}^*$ ,  $D_1\bar{D}^*$  or  $D_2\bar{D}^*$  do not decay into  $J/\psi\pi^+\pi^-$  if the pion pair comes from  $\sigma$  or  $f(980)$  in the heavy quark symmetry limit [68]. In other words, the discovery mode  $J/\psi\pi^+\pi^-$  disfavors the molecule interpretation of  $Y(4260)$ .

The observation of these exotic XYZ states provides us a platform to investigate the possible multiquark states beyond the conventional quark model. The experimental measurement of their decay behavior may shed light on their underlying structures. Different decay patterns may reveal their different inner structures.

In the heavy quark symmetry limit, the heavy quark spin within a hadron can not be flipped in the decay process, which provides us a useful handle to investigate the decay behavior of hadrons containing heavy quarks. Within the molecule assumption, the decay patterns of  $Z_c(3900)$ ,  $Z_c(4025)$  and  $Z_b(10610)/Z_b(10650)$  were discussed via the spin rearrangement scheme under heavy quark symmetry [64]. The discussion on the selection rules in the di-meson molecules was performed in Ref. [100]. Voloshin et al derived useful relations between the rate of the radiative transitions from  $\Upsilon(5S)$  to the hypothetical isovector molecular bottomonium resonance with negative G-parity using the spin rearrangement scheme [101]. An extensive investigations of the decay patterns of the hidden-charm molecular states with various quantum number can be found in Refs. [67, 68].

In this work we extend the spin rearrangement scheme to investigate the decay behavior of the XYZ states as the tetraquark candidates. We present the results of the hidden-charm decay patterns of the tetraquark states in the text since the expressions are slightly simpler. The decay matrix elements of their open charm decays contain many terms. In order to avoid the complicated and lengthy expressions, we collect the spin configurations of various open charm final states in Appendix D. One can check the spin configurations of the tetraquark states in Appendix A and different open charm final states in Appendix D. If the initial and final states have one or more common spin configurations, such a strong mode is allowed under the heavy quark symmetry. Otherwise, such a decay is suppressed.

One can compare the experimental decay patterns of the XYZ states with the theoretical predictions within the conventional quark model, the molecular and tetraquark schemes to determine the underlying structures of the XYZ states.

This paper is organized as follows. After the introduction, we give the general expressions about the color and spin structures of the tetraquarks in the heavy quark limit in Sec. II. We present the decay patterns of the S-wave tetraquarks in Sec. III. And we list the decay patterns of the P-wave tetraquarks in Sec. IV. In Sec. V, we focus on  $Y(4260)$ . We discuss the decay patterns of  $X(3872)$ ,  $Z_c(3900)$  etc in Sec. VI. The last section is the summary. The lengthy expressions and definitions are collected in Appendix A, B and C.

## II. THE FORMALISM

In the heavy quark limit, the spin-dependent chromomagnetic interaction proportional to  $1/m_Q$  is suppressed, while the chromoelectric interaction is spin-independent and dominant. For a hadron that contains heavy quarks, the total spin of heavy quarks  $S_H$  which is named as the "heavy spin", and the total angular momentum  $J$  are conserved. The sum of all the angular momenta other than the heavy spin is referred to the "light spin", which is also conserved with the definition  $\vec{S}_l \equiv \vec{J} - \vec{S}_H$ . The "heavy spin" and "light spin" provide an effective tool in the study of the structures of hadrons containing heavy quarks.

Considering a hidden charm  $cq\bar{c}\bar{q}$  tetraquark, the interaction Hamiltonian derived from the one-gluon exchange in the MIT bag model reads

$$H_{eff} = \sum_i m_i - \sum_{i>j} f_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (1)$$

where  $m_i$  refers to the  $i$ -th constituent quark mass,  $\vec{\lambda}_i$  and  $\vec{\sigma}_i$  are the color and spin operator respectively,  $f_{ij}$  are the coefficients depending on the specific models and systems [102–105]. The second term is the color-spin interaction which satisfies the  $SU(6)$  symmetry. For the anti-quarks, we have  $\vec{\lambda}_q = -\vec{\lambda}_q^*$  and  $\vec{\sigma}_q = -\vec{\sigma}_q^*$ . In general,  $f_{ij}$  have different values for different systems [106]. Here we use the spin rearrangement scheme to probe the strong decay behavior of the hidden charm tetraquarks. The specific values of  $f_{ij}$  are irrelevant in this work.

The eigenstates of the effective Hamiltonian in Eq. (1) can be chosen as  $|D_{6cs}, D_{3c}, J, N\rangle$ , where  $D_{6cs}$  and  $D_{3c}$  are the  $SU(6)_{cs}$  color-spin and  $SU(3)_c$  color representations of the multiquark system respectively.  $J$  is the total angular momentum and  $N$  is the total number of quarks and antiquarks. We want to emphasize that the following discussions also hold for the general interaction in Eq. (27).

We employ the diquark models to construct the color spin wave functions of the hidden charm tetraquarks, which are denoted as  $|cq, D_{6cs}, D_{3c}, J, 2\rangle \otimes |\bar{c}\bar{q}, D_{6cs}, D_{3c}, J, 2\rangle$ . The  $SU(6)_{cs}$  color-spin representations of the diquark are  $6_{cs} \otimes 6_{cs} = 21_{cs} \oplus \bar{15}_{cs}$ . For the anti-diquark, we have  $\bar{6}_{cs} \otimes \bar{6}_{cs} = \bar{21}_{cs} \oplus 15_{cs}$ . The  $SU(3)_c$  color representations of the diquark are  $3_c \otimes 3_c = 6_c \oplus \bar{3}_c$ , and the anti-diquark parts are  $\bar{3}_c \otimes \bar{3}_c = \bar{6}_c \oplus 3_c$ . The diquark couples with the anti-diquark to form a tetraquark,

$$\begin{aligned} 21_{cs} \otimes \bar{21}_{cs} &= 405_{cs} \oplus 280_{cs} \oplus 35_{cs} \oplus 1_{cs} \\ 21_{cs} \otimes 15_{cs} &= 280_{cs} \oplus 35_{cs} \\ 15_{cs} \otimes \bar{15}_{cs} &= 189_{cs} \oplus 35_{cs} \oplus 1_{cs}. \end{aligned}$$

The color-spin wave functions of the S-wave hidden charm tetraquarks with the quantum numbers  $J^P = 0^+, 1^+, 2^+$  are

$$\begin{aligned}
|1_{cs}, 1_c, 0, 4\rangle &= \sqrt{\frac{6}{7}}|cq, 21_{cs}, 6_c, 1, 2\rangle|\bar{c}\bar{q}, \bar{2}1_{cs}, \bar{6}_c, 1, 2\rangle + \sqrt{\frac{1}{7}}|cq, 21_{cs}, \bar{3}_c, 0, 2\rangle|\bar{c}\bar{q}, \bar{2}1_{cs}, 3_c, 0, 2\rangle \\
|405_{cs}, 1_c, 0, 4\rangle &= \sqrt{\frac{1}{7}}|cq, 21_{cs}, 6_c, 1, 2\rangle|\bar{c}\bar{q}, \bar{2}1_{cs}, \bar{6}_c, 1, 2\rangle - \sqrt{\frac{6}{7}}|cq, 21_{cs}, \bar{3}_c, 0, 2\rangle|\bar{c}\bar{q}, \bar{2}1_{cs}, 3_c, 0, 2\rangle \\
|1_{cs}, 1_c, 0, 4\rangle &= \sqrt{\frac{3}{5}}|cq, 15_{cs}, \bar{3}_c, 1, 2\rangle|\bar{c}\bar{q}, \bar{1}5_{cs}, 3_c, 1, 2\rangle + \sqrt{\frac{2}{5}}|cq, 15_{cs}, 6_c, 0, 2\rangle|\bar{c}\bar{q}, \bar{1}5_{cs}, \bar{6}_c, 0, 2\rangle \\
|189_{cs}, 1_c, 0, 4\rangle &= \sqrt{\frac{2}{5}}|cq, 15_{cs}, \bar{3}_c, 1, 2\rangle|\bar{c}\bar{q}, \bar{1}5_{cs}, 3_c, 1, 2\rangle - \sqrt{\frac{3}{5}}|cq, 15_{cs}, 6_c, 0, 2\rangle|\bar{c}\bar{q}, \bar{1}5_{cs}, \bar{6}_c, 0, 2\rangle, \\
|35_{cs}, 1_c, 1, 4\rangle &= |cq, 21_{cs}, 6_c, 1, 2\rangle|\bar{c}\bar{q}, \bar{2}1_{cs}, \bar{6}_c, 1, 2\rangle \\
|35_{cs}, 1_c, 1, 4\rangle &= |cq, 15_{cs}, \bar{3}_c, 1, 2\rangle|\bar{c}\bar{q}, \bar{1}5_{cs}, 3_c, 1, 2\rangle, \\
|35_{cs}, 1_c, 1, 4\rangle &= \sqrt{\frac{1}{3}}|cq, 21_{cs}, \bar{3}_c, 0, 2\rangle|\bar{c}\bar{q}, \bar{1}5_{cs}, 3_c, 1, 2\rangle - \sqrt{\frac{2}{3}}|cq, 21_{cs}, 6_c, 1, 2\rangle|\bar{c}\bar{q}, \bar{1}5_{cs}, \bar{6}_c, 0, 2\rangle \\
|280_{cs}, 1_c, 1, 4\rangle &= \sqrt{\frac{2}{3}}|cq, 21_{cs}, \bar{3}_c, 0, 2\rangle|\bar{c}\bar{q}, \bar{1}5_{cs}, 3_c, 1, 2\rangle + \sqrt{\frac{1}{3}}|cq, 21_{cs}, 6_c, 1, 2\rangle|\bar{c}\bar{q}, \bar{1}5_{cs}, \bar{6}_c, 0, 2\rangle \\
|35_{cs}, 1_c, 1, 4\rangle &= \sqrt{\frac{1}{3}}|cq, 15_{cs}, \bar{3}_c, 1, 2\rangle|\bar{c}\bar{q}, \bar{2}1_{cs}, 3_c, 0, 2\rangle - \sqrt{\frac{2}{3}}|cq, 15_{cs}, 6_c, 0, 2\rangle|\bar{c}\bar{q}, \bar{2}1_{cs}, \bar{6}_c, 1, 2\rangle \\
|280_{cs}, 1_c, 1, 4\rangle &= \sqrt{\frac{2}{3}}|cq, 15_{cs}, \bar{3}_c, 1, 2\rangle|\bar{c}\bar{q}, \bar{2}1_{cs}, 3_c, 0, 2\rangle + \sqrt{\frac{1}{3}}|cq, 15_{cs}, 6_c, 0, 2\rangle|\bar{c}\bar{q}, \bar{2}1_{cs}, \bar{6}_c, 1, 2\rangle, \\
|405_{cs}, 1_c, 2, 4\rangle &= |cq, 21_{cs}, 6_c, 1, 2\rangle|\bar{c}\bar{q}, \bar{2}1_{cs}, \bar{6}_c, 1, 2\rangle \\
|189_{cs}, 1_c, 2, 4\rangle &= |cq, 15_{cs}, \bar{3}_c, 1, 2\rangle|\bar{c}\bar{q}, \bar{1}5_{cs}, 3_c, 1, 2\rangle.
\end{aligned}$$

Besides the heavy and light spin of a hadron, the isospin is another conserved quantity in the strong decays. The hidden charm tetraquarks  $cq\bar{c}\bar{q}$  can be categorized as the isovector and isoscalar states with the corresponding color-spin wave functions

$$\begin{aligned}
|cq\bar{c}\bar{q}\rangle_1^+ &= |cu\rangle_{(D_{6cs}, D_{3c}, J)} \otimes |\bar{c}\bar{d}\rangle_{(D'_{6cs}, D'_{3c}, J')} \\
|cq\bar{c}\bar{q}\rangle_1^0 &= \frac{1}{\sqrt{2}}[|cd\rangle_{(D_{6cs}, D_{3c}, J)} \otimes |\bar{c}\bar{d}\rangle_{(D'_{6cs}, D'_{3c}, J')} - |cu\rangle_{(D_{6cs}, D_{3c}, J)} \otimes |\bar{c}\bar{u}\rangle_{(D'_{6cs}, D'_{3c}, J')}] \\
|cq\bar{c}\bar{q}\rangle_1^- &= |cd\rangle_{(D_{6cs}, D_{3c}, J)} \otimes |-\bar{c}\bar{u}\rangle_{(D'_{6cs}, D'_{3c}, J')}, \\
|cq\bar{c}\bar{q}\rangle_1^0 &= \frac{1}{\sqrt{2}}[|cd\rangle_{(D_{6cs}, D_{3c}, J)} \otimes |\bar{c}\bar{d}\rangle_{(D'_{6cs}, D'_{3c}, J')} + |cu\rangle_{(D_{6cs}, D_{3c}, J)} \otimes |\bar{c}\bar{u}\rangle_{(D'_{6cs}, D'_{3c}, J')}]
\end{aligned}$$

The heavy quark spin is not flipped in the strong decays. The heavy spin, light spin, total angular momentum and isospin are conserved quantities. For a neutral hidden charm system, its G-parity and C-parity are also conserved. We define the C-parity eigenstates for the neutral partners of the hidden charm tetraquarks following the standard convention

$$\frac{1}{\sqrt{2}}\left\{[(cq]_{(d_6, d_3, S)} \otimes [\bar{c}\bar{q}]_{(d'_6, d'_3, S')}]_{(D_6, D_3, K)} \otimes L\right\}_{(D_6, D_3, J)} \pm \left\{[(cq]_{(d'_6, d'_3, S')} \otimes [\bar{c}\bar{q}]_{(d_6, d_3, S)}]_{(D_6, D_3, K)} \otimes L\right\}_{(D_6, D_3, J)},$$

where  $S$ ,  $d_6$  and  $d_3$  denote the total spin,  $SU(6)_{cs}$  color-spin representations and  $SU(3)_c$  color representations of the diquark  $[cq]$ , and  $S'$ ,  $d'_6$  and  $d'_3$  for the anti-diquarks  $[\bar{c}\bar{q}]$ .  $K$  represents the total spin of the four quarks.  $L$  is the orbital angular momenta within the tetraquark.  $J$ ,  $D_6$  and  $D_3$  denote the total spin,  $SU(6)_{cs}$  color-spin and  $SU(3)_c$  color representations of tetraquarks  $[cq\bar{c}\bar{q}]$ . The  $\pm$  corresponds to C-parity  $C \doteq \pm$ .

The possible  $J^{PC}$  quantum numbers of the neutral partners of a hidden charm tetraquark without any orbital excitation are  $0^{++}$ ,  $1^{+-}$ ,  $1^{++}$  and  $2^{++}$ . Within the framework of the spin rearrangement scheme, the total spin can be recoupled from the total spin of the heavy quark pair  $c\bar{c}$  and the total spin of the light quark pair  $q\bar{q}$ . If the total spin of either the heavy quark pair or light quark pair is 1 and the total spin of the other quark pair is zero, the total spin of the tetraquark is 1 while it has negative C-parity. If both the heavy quark pair and light quark pair have the total spin 1, the total spin of the tetraquark can also be 1 but it has

positive C-parity. The S-wave tetraquark can carry either positive or negative C-parity when its total spin is 1. This argument can be illustrated by the expanded color-spin structures in the following.

If there is a P-wave excitation within the hidden charm tetraquark, its parity is negative. Now the C-parity of the neutral partner of a P-wave hidden charm tetraquark is rather complicated. The charm quark  $c$  and the light quark  $q$  constitute the diquark  $[cq]$ ,  $\bar{c}$  and  $\bar{q}$  constitute the anti-diquark  $[\bar{c}\bar{q}]$ . If the P-wave excitation exists between the charm quark and light quark in the diquark (or anti-diquark), the P-wave excitation does not correlate with the charge conjugation transformation and does not contribute to its  $C$  parity. However, if the P-wave excitation exists between the diquark and anti-quark pair, the  $C$  transformation and P-wave excitation are correlated with each other, which is the same as in the neutral meson case where  $C = (-)^{L+S}$ . If one treats the antidiquark and diquark as an effective quark and antiquark, one arrives at the same C-parity formula when there exists an orbital excitation between the diquark and antidiquark pair. In other words, the different location of the P-wave excitation affects the C-parity of the tetraquark. We need to distinguish these two kinds of the P-wave excitations and give an elaborate discussion in the next section.

The decomposition of the total spin of the above tetraquarks into the heavy spin and light spin will cast light on their decay behavior. We employ the color-spin re-coupling formula in analyzing the general color-spin structure as in Ref. [106]. For instance, a physical hidden charm tetraquark must be singlet in color  $SU(3)_c$  presentation. Its color-spin structures can be written as

$$\begin{aligned} & \left\{ \left[ [c_1 q_2 D_{6cs}^{12} D_{3c}^{12} S^{12}] \otimes [\bar{c}_3 \bar{q}_4 D_{6cs}^{34} D_{3c}^{34} S^{34}] \right]_{(D_{6cs}, K)} \otimes L \right\}_{(D_{6cs}, J)} \\ &= \sum_{D_{3c}^{13}, D_{3c}^{24}} R_c[D_{3c}^{12}, D_{3c}^{34}, D_{3c}^{13}, D_{3c}^{24}, D_{6cs}] \sum_{S^{24}, S_H, S_l} R_s[S^{12}, S^{34}, K, L; S^{24}, S_H, S_l; J] \\ & \times \left\{ \left[ c_1 \bar{c}_3 D_{6cs}^{13} D_{3c}^{13} S_H \right] \otimes \left[ ([q_2 \bar{q}_4 S^{24}] \otimes L) D_{6cs}^{24} D_{3c}^{24} S_l \right] \right\}_{(D_{6cs}, J)}, \end{aligned}$$

where the color-recoupling coefficients  $R_c$  are

$$\begin{aligned} & R_c[(\lambda^{12} \mu^{12})(\lambda^{34} \mu^{34}); (\lambda^{13} \mu^{13})(\lambda^{24} \mu^{24})] \\ &= (-1)^{\lambda^{12} + \mu^{12} + \lambda^{13} + \mu^{13}} U[(10)(10)(10)(01); (\lambda^{12} \mu^{12})(\lambda^{13} \mu^{13})], \end{aligned}$$

with  $SU(3)_c$  Racah coefficients  $U$  which were discussed in Refs. [107, 108]. The angular momentum recoupling coefficients  $R_s$  are

$$\begin{aligned} & R_s[S^{12}, S^{34}, K, L; S^{24}, S_H, S_l; J] \\ &= (-1)^{S_H + S^{24} + L + J} \sqrt{(2S^{12} + 1)(2S^{34} + 1)(2S^{13} + 1)(2S_H + 1)(2S_l + 1)(2K + 1)} \\ & \times \left\{ \begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & S^{12} \\ \frac{1}{2} & \frac{1}{2} & S^{34} \\ S_H & S^{24} & K \end{array} \right\} \left\{ \begin{array}{ccc} S^{24} & S_H & K \\ J & L & S_l \end{array} \right\}, \end{aligned}$$

where  $q_2$  and  $q_4$  are the light quarks  $u$  and  $d$ . We use the subscripts  $H$  and  $l$  to distinguish the heavy and light spins of a tetraquark.  $S_H$  and  $S_l$  denote the heavy and light spins after the color-spin rearrangement. The notation  $\{[c_1 \bar{c}_3 D_{6cs}^{13} D_{3c}^{13} S_H] \otimes [(q_2 \bar{q}_4 S^{24}] \otimes L) D_{6cs}^{24} D_{3c}^{24} S_l\}_{(D_{6cs}, J)}$  means that the spins of the  $c$  and  $\bar{c}$  quarks are coupled into the heavy quark spin  $S_H$  while the spins of the  $q$  and  $\bar{q}$  quarks are coupled into the light quark spin  $S^{24}$ . And then  $S^{24}$  couples with the orbital angular momentum  $L$  to form the light spin  $S_l$ . The notations  $D_{6cs}^{ij}$  and  $D_{3c}^{ij}$  denote the color-spin and color representations of the corresponding parts.

Applying the conventional definition for the C-parity eigenstates, we can express the neutral parts of the tetraquarks and its isospin partners as

$$\begin{aligned} |cq\bar{c}\bar{q}\rangle_1^+ &= \frac{1}{\sqrt{2}} \left\{ \left[ ([cu]_{d_6 d_3 S} [\bar{c}\bar{d}]_{d'_6 d'_3 S'})_{D_6 K} \otimes L \right]_{D_6 J} + \tilde{c} \left[ ([cu]_{d'_6 d'_3 S'} [\bar{c}\bar{d}]_{d_6 d_3 S})_{D_6 K} \otimes L \right]_{D_6 J} \right\} \\ |cq\bar{c}\bar{q}\rangle_1^0 &= \frac{1}{2} \left\{ \left[ ([cd]_{d_6 d_3 S} [\bar{c}\bar{d}]_{d'_6 d'_3 S'} - [cu]_{d_6 d_3 S} [\bar{c}\bar{u}]_{d'_6 d'_3 S'})_{D_6 K} \otimes L \right]_{D_6 J} \right. \\ & \quad \left. + \tilde{c} \left[ ([cd]_{d'_6 d'_3 S'} [\bar{c}\bar{d}]_{d_6 d_3 S} - [cu]_{d'_6 d'_3 S'} [\bar{c}\bar{u}]_{d_6 d_3 S})_{D_6 K} \otimes L \right]_{D_6 J} \right\} \\ |cq\bar{c}\bar{q}\rangle_1^- &= \frac{1}{\sqrt{2}} \left\{ \left[ ([cd]_{d_6 d_3 S} [-\bar{c}\bar{u}]_{d'_6 d'_3 S'})_{D_6 K} \otimes L \right]_{D_6 J} + \tilde{c} \left[ ([cd]_{d'_6 d'_3 S'} [-\bar{c}\bar{u}]_{d_6 d_3 S})_{D_6 K} \otimes L \right]_{D_6 J} \right\}, \end{aligned}$$

and the isoscalar part of the hidden charm tetraquarks can be rewritten as

$$|cq\bar{c}\bar{q}\rangle_1^0 = \frac{1}{2} \left\{ \left[ ([cd]_{d_6 d_3 S} [\bar{c}\bar{d}]_{d'_6 d'_3 S'} + [cu]_{d_6 d_3 S} [\bar{c}\bar{u}]_{d'_6 d'_3 S'})_{D_6 K} \otimes L \right]_{D_6 J} \right. \\ \left. + \tilde{c} \left[ ([cd]_{d'_6 d'_3 S'} [\bar{c}\bar{d}]_{d_6 d_3 S} + [cu]_{d'_6 d'_3 S'} [\bar{c}\bar{u}]_{d_6 d_3 S})_{D_6 K} \otimes L \right]_{D_6 J} \right\},$$

where the coefficient  $\tilde{c}$  is +1 or -1 for the positive or negative C-parity respectively. The subscripts  $d_6^{(\prime)}$ ,  $d_3^{(\prime)}$  and  $S^{(\prime)}$  denote the color-spin representations, color representations and total spins of corresponding parts.  $D_6$  represent the color-spin representations of the total systems.  $K$ ,  $L$  and  $J$  denote the total spin of the four quarks, the orbital angular momenta inside the system and the total angular momentum respectively.

With the color-spin rearrangement scheme, the total angular momentum and  $SU(3)_c$  color singlet of the system are decomposed into heavy spin, light spin and their corresponding  $SU(3)_c$  color representations. The color-spin structures of the isovector states after recoupling can be obtained as

$$|cq\bar{c}\bar{q}\rangle_1^+ = \frac{1}{\sqrt{2}} \left\{ \sum R_c R_s \left[ [c\bar{c}]_{D_6^{13} \bar{D}_3^{13} S_H} \otimes ([u\bar{d}]_{S^{24}} \otimes L)_{D_6^{24} D_3^{24} S_I} \right]_{D_6 J} |(cu)_{d_6 d_3 S} (\bar{c}\bar{d})_{d'_6 d'_3 S'} \rangle \right. \\ \left. + \tilde{c} \sum R_c R_s \left[ [c\bar{c}]_{\bar{D}_6^{13} \bar{D}_3^{13} \bar{S}_H} \otimes ([u\bar{d}]_{\bar{S}^{24}} \otimes L)_{\bar{D}_6^{24} \bar{D}_3^{24} \bar{S}_I} \right]_{D_6 J} |(cu)_{d'_6 d'_3 S'} (\bar{c}\bar{d})_{d_6 d_3 S} \rangle \right\} \quad (2)$$

$$|cq\bar{c}\bar{q}\rangle_1^0 = \frac{1}{2} \left\{ \sum R_c R_s \left[ [c\bar{c}]_{D_6^{13} \bar{D}_3^{13} S_H} \otimes \left( \left[ \frac{d\bar{d} - u\bar{u}}{\sqrt{2}} \right]_{S^{24}} \otimes L \right)_{D_6^{24} D_3^{24} S_I} \right]_{D_6 J} \right. \\ \times \left( \frac{|(cd)_{d_6 d_3 S} (\bar{c}\bar{d})_{d'_6 d'_3 S'} \rangle - |(cu)_{d_6 d_3 S} (\bar{c}\bar{u})_{d'_6 d'_3 S'} \rangle}{\sqrt{2}} \right) \\ \left. + \tilde{c} \sum R_c R_s \left[ [c\bar{c}]_{\bar{D}_6^{13} \bar{D}_3^{13} \bar{S}_H} \otimes \left( \left[ \frac{d\bar{d} - u\bar{u}}{\sqrt{2}} \right]_{\bar{S}^{24}} \otimes L \right)_{\bar{D}_6^{24} \bar{D}_3^{24} \bar{S}_I} \right]_{D_6 J} \right. \\ \left. \times \left( \frac{|(cd)_{d'_6 d'_3 S'} (\bar{c}\bar{d})_{d_6 d_3 S} \rangle - |(cu)_{d'_6 d'_3 S'} (\bar{c}\bar{u})_{d_6 d_3 S} \rangle}{\sqrt{2}} \right) \right\} \quad (3)$$

$$|cq\bar{c}\bar{q}\rangle_1^- = \frac{1}{\sqrt{2}} \left\{ \sum R_c R_s \left[ [c\bar{c}]_{D_6^{13} \bar{D}_3^{13} S_H} \otimes ([-d\bar{u}]_{S^{24}} \otimes L)_{D_6^{24} D_3^{24} S_I} \right]_{D_6 J} |-(cd)_{d_6 d_3 S} (\bar{c}\bar{u})_{d'_6 d'_3 S'} \rangle \right. \\ \left. + \tilde{c} \sum R_c R_s \left[ [c\bar{c}]_{\bar{D}_6^{13} \bar{D}_3^{13} \bar{S}_H} \otimes ([-d\bar{u}]_{\bar{S}^{24}} \otimes L)_{\bar{D}_6^{24} \bar{D}_3^{24} \bar{S}_I} \right]_{D_6 J} |-(cd)_{d'_6 d'_3 S'} (\bar{c}\bar{u})_{d_6 d_3 S} \rangle \right\}. \quad (4)$$

Similarly, the isoscalar partner of the states can be expanded as

$$|cq\bar{c}\bar{q}\rangle_1^0 = \frac{1}{2} \left\{ \sum R_c R_s \left[ [c\bar{c}]_{D_6^{13} \bar{D}_3^{13} S_H} \otimes \left( \left[ \frac{d\bar{d} + u\bar{u}}{\sqrt{2}} \right]_{S^{24}} \otimes L \right)_{D_6^{24} D_3^{24} S_I} \right]_{D_6 J} \right. \\ \times \left( \frac{|(cd)_{d_6 d_3 S} (\bar{c}\bar{d})_{d'_6 d'_3 S'} \rangle + |(cu)_{d_6 d_3 S} (\bar{c}\bar{u})_{d'_6 d'_3 S'} \rangle}{\sqrt{2}} \right) \\ \left. + \tilde{c} \sum R_c R_s \left[ [c\bar{c}]_{\bar{D}_6^{13} \bar{D}_3^{13} \bar{S}_H} \otimes \left( \left[ \frac{d\bar{d} + u\bar{u}}{\sqrt{2}} \right]_{\bar{S}^{24}} \otimes L \right)_{\bar{D}_6^{24} \bar{D}_3^{24} \bar{S}_I} \right]_{D_6 J} \right. \\ \left. \times \left( \frac{|(cd)_{d'_6 d'_3 S'} (\bar{c}\bar{d})_{d_6 d_3 S} \rangle + |(cu)_{d'_6 d'_3 S'} (\bar{c}\bar{u})_{d_6 d_3 S} \rangle}{\sqrt{2}} \right) \right\}. \quad (5)$$

In Eqs. (2)-(5), we have explicitly included the flavor wave function, for instance,  $|(cu)_{d_6 d_3 S} (\bar{c}\bar{d})_{d'_6 d'_3 S'} \rangle$ . Here we use the color-spin, color representations and the total spin of diquark or anti-diquark to distinguish different flavor but with the same constituents and total representations. If the tetraquarks have the same constituents and total representations but their inner diquarks or anti-diquarks have the different representations, these tetraquarks should be different physical states. The orthogonalization of these different states are guaranteed when we label their flavor wave functions with the color-spin, color representations and the total spin of their diquark or anti-diquark.

In order to probe the decay pattern of the hidden charm tetraquarks under heavy quark symmetry, we also decompose the

charmonium final states into the heavy spin and light spin, which read

$$|\eta_c(1^1S_0)\rangle = |(0_H^- \otimes 0_l^+)_{0^+}^+|(c\bar{c})_{1_{cs}1_c0}\rangle, \quad (6)$$

$$|J/\psi(1^3S_1)\rangle = |(1_H^- \otimes 0_l^+)_{1^+}^+|(c\bar{c})_{35_{cs}1_c1}\rangle, \quad (7)$$

$$|h_c(1^1P_1)\rangle = |(0_H^- \otimes 1_l^-)_{1^+}^+|(c\bar{c})_{1_{cs}1_c1}\rangle, \quad (8)$$

$$|\chi_{c0}(1^3P_0)\rangle = |(1_H^- \otimes 1_l^-)_{0^+}^+|(c\bar{c})_{35_{cs}1_c0}\rangle, \quad (9)$$

$$|\chi_{c1}(1^3P_1)\rangle = |(1_H^- \otimes 1_l^-)_{1^+}^+|(c\bar{c})_{35_{cs}1_c1}\rangle, \quad (10)$$

$$|\chi_{c2}(1^3P_2)\rangle = |(1_H^- \otimes 1_l^-)_{2^+}^+|(c\bar{c})_{35_{cs}1_c2}\rangle, \quad (11)$$

$$|\eta_{c2}(1^1D_2)\rangle = |(0_H^- \otimes 2_l^+)_{2^+}^+|(c\bar{c})_{1_{cs}1_c2}\rangle, \quad (12)$$

$$|\psi(1^3D_1)\rangle = |(1_H^- \otimes 2_l^+)_{1^+}^+|(c\bar{c})_{35_{cs}1_c1}\rangle, \quad (13)$$

$$|\psi(1^3D_2)\rangle = |(1_H^- \otimes 2_l^+)_{2^+}^+|(c\bar{c})_{35_{cs}1_c2}\rangle, \quad (14)$$

$$|\psi(1^3D_3)\rangle = |(1_H^- \otimes 2_l^+)_{3^+}^+|(c\bar{c})_{35_{cs}1_c3}\rangle. \quad (15)$$

Following our previous work, the flavor wave function is defined as  $|(c\bar{c})\rangle \equiv \frac{1}{\sqrt{2}}(|\bar{c}c\rangle + |c\bar{c}\rangle)$  [67, 68]. The superscripts + and - inside the parentheses represent the positive and negative parity of the corresponding parts respectively, while the superscripts -+ outside the parentheses correspond to the quantum numbers  $PC$ . The subscripts 0, 1, 2, 3 outside the parentheses denote the total angular momentum  $J$  of the charmonium. The subscripts outside in the flavor wave function have the same meaning as in Eqs. (6)-(15). The  $C$  parity of the charmonium is reflected in the spin wave functions, i.e.,  $C = (-1)^{S_H+S_l}$ . Obviously, the color representation of those observed charmonium is singlet.

We also introduce the color-spin structures of the light mesons,

$$|\pi^+\rangle = |(0_H^+ \otimes 0_l^-)_{0^+}^+|(u\bar{d})_{1_{cs}1_c0}\rangle, \quad (16)$$

$$|\pi^0\rangle = |(0_H^+ \otimes 0_l^-)_{0^+}^+|\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})_{1_{cs}1_c0}\rangle, \quad (17)$$

$$|\pi^-\rangle = |(0_H^+ \otimes 0_l^-)_{0^+}^+|-(d\bar{u})_{1_{cs}1_c0}\rangle, \quad (18)$$

$$|\rho^+\rangle = |(0_H^+ \otimes 1_l^-)_{1^+}^+|(u\bar{d})_{35_{cs}1_c1}\rangle, \quad (19)$$

$$|\rho^0\rangle = |(0_H^+ \otimes 1_l^-)_{1^+}^+|\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})_{35_{cs}1_c1}\rangle, \quad (20)$$

$$|\rho^-\rangle = |(0_H^+ \otimes 1_l^-)_{1^+}^+|-(d\bar{u})_{35_{cs}1_c1}\rangle, \quad (21)$$

$$|\eta\rangle = |(0_H^+ \otimes 0_l^-)_{0^+}^+|\frac{1}{\sqrt{2}}(d\bar{d} + u\bar{u})_{1_{cs}1_c0}\rangle, \quad (22)$$

$$|\omega\rangle = |(0_H^+ \otimes 1_l^-)_{1^+}^+|\frac{1}{\sqrt{2}}(d\bar{d} + u\bar{u})_{35_{cs}1_c1}\rangle, \quad (23)$$

$$|\sigma\rangle = |(0_H^+ \otimes 0_l^+)_{0^+}^+|\frac{1}{\sqrt{2}}(d\bar{d} + u\bar{u})_{1_{cs}1_c0}\rangle. \quad (24)$$

The orthogonalization of the color-spin wave functions leads to

$$\begin{aligned} & \left\langle \left( a_{D_6^H D_3^H}^H \otimes b_{D_6^L D_3^L}^L \right)_{D_6 D_3 J}^{pc} \left| \left( c_{\bar{D}_6^H \bar{D}_3^H}^H \otimes d_{\bar{D}_6^L \bar{D}_3^L}^L \right)_{\bar{D}_6 \bar{D}_3 J'}^{p'c'} \right. \right\rangle \\ &= \delta_{ac} \delta_{bd} \delta_{JJ'} \delta_{pp'} \delta_{cc'} \delta_{D_6 D_6'} \delta_{D_3 D_3'} \delta_{D_6^H \bar{D}_6^H} \delta_{D_3^H \bar{D}_3^H} \delta_{D_6^L \bar{D}_6^L} \delta_{D_3^L \bar{D}_3^L}, \end{aligned}$$

where the superscripts  $p^{(\prime)}$  and  $c^{(\prime)}$  represent the parity and  $C$  parity, respectively. The superscripts  $H$  and  $L$  denote the heavy spin part and light spin part.  $D_6^H$  and  $\bar{D}_6^H$  represent the color-spin  $S U(6)_{cs}$  representations of the heavy quarks. Similarly,  $D_6^L$  and  $\bar{D}_6^L$  represent the color-spin  $S U(6)_{cs}$  representations of the light quark.  $D_3^H$  and  $\bar{D}_3^H$  denote the color  $S U(3)_c$  representations of the heavy quarks. Likewise,  $D_3^L$  and  $\bar{D}_3^L$  denote the color  $S U(3)_c$  representations of the light quarks. The subscripts  $D_6^{(\prime)}$  and  $D_3^{(\prime)}$  represent the color-spin  $S U(6)_{cs}$  representations and the color  $S U(3)_3$  representations of the whole systems respectively.

Moreover, the orthogonalization of the flavor wave functions are defined as

$$\begin{aligned} & \left\langle (cq_i)_{\bar{d}_6 \bar{d}_3 \bar{S}} (\bar{c}\bar{q}_m)_{\bar{d}'_6 \bar{d}'_3 \bar{S}'} \left| (cq_j)_{d_6 d_3 S} (\bar{c}\bar{q}_n)_{d'_6 d'_3 S'} \right. \right\rangle \\ &= \delta_{ij} \delta_{mn} \delta_{\bar{d}_6 d'_6} \delta_{\bar{d}_3 d'_3} \delta_{\bar{S} S'} \delta_{\bar{d}'_6 d'_6} \delta_{\bar{d}'_3 d'_3} \delta_{\bar{S}' S'}, \end{aligned}$$



where  $q_i, q_j, q_m$  and  $q_n$  can be  $u$  or  $d$  quark. We need to specify that the ordering of the  $\bar{c}$  and  $c, q_i$  and  $\bar{q}_m$  can not be interchanged [67, 68]. The above definition guarantees the orthogonalization of their total wave functions.

In the heavy quark symmetry limit, the effective Hamiltonian  $H_{eff}$  for a decay process can be separated into the spatial and flavor parts,

$$H_{eff} = H_{eff}^{spatial} \otimes H_{eff}^{flavor}, \quad (25)$$

In the strong decays, the effective Hamiltonian  $H_{eff}$  conserves the heavy spin, light spin, isospin, parity, C parity and  $G$ -parity separately, while it may change the color-spin  $SU(6)_{cs}$  and color  $SU(3)_c$  representations. The general form of the decay matrix can be expressed in terms of the reduced matrix,

$$\begin{aligned} & \left\langle \left( a_{D_6^H D_3^H}^H \otimes b_{D_6^L D_3^L}^L \right)_{D_6 D_3 J}^{pc} \left| H_{eff}^{spatial} \left( c_{\bar{D}_6^H \bar{D}_3^H}^H \otimes d_{\bar{D}_6^L \bar{D}_3^L}^L \right)_{\bar{D}_6' \bar{D}_3' J'}^{p'c'} \right\rangle \right. \\ &= \delta_{ac} \delta_{bd} \delta_{JJ'} \delta_{pp'} \delta_{cc'} \left\langle \phi_{D_6^H D_3^H; D_6^L D_3^L}^{D_6 D_3} \left\| H_{eff}^{spatial} \left\| \phi_{\bar{D}_6^H \bar{D}_3^H; \bar{D}_6^L \bar{D}_3^L}^{D_6' D_3'} \right\| \right\rangle, \end{aligned}$$

where  $\phi$  in the reduced matrix denotes the radial wave functions, which may depend on the different  $SU(6)_{cs}$  and  $SU(3)_c$  representations. This formula reflects the conservation of the parity, C parity, the total angular momentum, heavy spin, and light spin. In addition, the flavor part of the decay matrix can be written as

$$\left\langle (cq_i)_{\bar{d}_6 \bar{d}_3 \bar{S}} (\bar{c}\bar{q}_m)_{\bar{d}'_6 \bar{d}'_3 \bar{S}'} \left| H_{eff}^{flavor} \right| (cq_j)_{d_6 d_3 S} (\bar{c}\bar{q}_n)_{d'_6 d'_3 S'} \right\rangle = \delta_{ij} \delta_{mn}.$$

For the decays  $|cq\bar{c}\bar{q}\rangle \rightarrow (c\bar{c}) + \text{light meson}$ , the transition matrix elements related to the flavor wave functions can be written as

$$\left\langle (c\bar{c})_{\bar{d}_6 \bar{d}_3 \bar{S}} \left| \left( (q_i \bar{q}_m)_{\bar{d}'_6 \bar{d}'_3 \bar{S}'} \right) \left| H_{eff}^{flavor} \right| (cq_j)_{d_6 d_3 S} (\bar{c}\bar{q}_n)_{d'_6 d'_3 S'} \right\rangle = \delta_{ij} \delta_{mn}.$$

We also need the rearranged color-spin structures of the final states. Its general expression reads

$$\begin{aligned} & | \text{charmonia} \rangle \otimes | \text{light meson} \rangle \\ &= \left[ [(c\bar{c})_{d_6 d_3 S_H} \otimes L]_K \otimes Q_{d'_6 d'_3 S_I} \right]_J | (c\bar{c})_{d_6 d_3 S} \rangle | (q_i \bar{q}_j)_{d'_6 d'_3 S'} \rangle \\ &= \sum_{S_I=|L-Q|}^{L+Q} \mathcal{D}_{S_H, S_I}^{Q, L, K, J} \left| \left[ (c\bar{c})_{d_6 d_3 S_H} \otimes [L \otimes Q]_{d'_6 d'_3 S_I} \right]_J \right\rangle | (c\bar{c})_{d_6 d_3 S} \rangle | (q_i \bar{q}_j)_{d'_6 d'_3 S'} \rangle, \end{aligned} \quad (26)$$

where the indices  $c, \bar{c}$  and  $q_i, \bar{q}_j$  in the square brackets denote the corresponding spin wave functions. And  $g$  and  $L$  denote the heavy and light spins of the charmonium, respectively. We collect the coefficients  $\mathcal{D}_{S_H, S_I}^{Q, L, K, J}$  in Table I.

### III. DECAY PATTERNS OF THE S-WAVE TETRAQUARKS

#### A. Color-spin structures of the S-wave tetraquarks

In this section we discuss the color-spin structures of the S-wave hidden charm tetraquarks in Eqs. (2)-(5), which are collected in Appendix A. After the color-spin rearrangement, the charm quark and anti-charm quark are coupled into the heavy spin, color-spin and color representations. We use the combination  $(1, 1, 0)_H^+ \otimes (1, 1, 0)_L^+$  to indicate that the  $c\bar{c}$  quarks and light quarks couple into the color-spin  $SU(6)_{cs}$  singlet, color  $SU(3)_c$  singlet and the heavy/light spin singlet respectively. Then both the heavy part and light part have the negative parity and the positive C-parity. The subscripts  $(21, \bar{21})$  in  $|1_{cs}, 1_c, 0\rangle_{(21, \bar{21})}$  in Eq. (32) indicate that its diquark has the color-spin representations 21 and the anti-diquark has the color-spin representations  $\bar{21}$ . We use the color-spin representations of the diquark and anti-diquark to distinguish the tetraquark states that have the same total representations and total spin.

There are four tetraquark states with  $J^{PC} = 0^{++}$ . Their isospin wave functions do not affect the results of the color-spin re-coupling. Both the isoscalar and isovector tetraquarks have the same color-spin wave functions, but have different flavor functions. All the four states contain the color  $SU(3)_c$  singlet and octet, which differs from the color-spin wave functions of the molecular states. In addition, all the four states include the heavy spin singlet and triplet. The color octet terms in the color-spin wave functions of the states  $|1_{cs}, 1_c, 0\rangle_{(21, \bar{21})}$  and  $|1_{cs}, 1_c, 0\rangle_{(15, \bar{15})}$  contain the heavy spin singlet and triplet. The color octet configurations of the states  $|405_{cs}, 1_c, 0\rangle_{(21, \bar{21})}$  and  $|189_{cs}, 1_c, 0\rangle_{(15, \bar{15})}$  contain the heavy spin singlet only.

TABLE I: The coefficient  $\mathcal{D}_{S_H S_L}^{Q, L, K, J}$  in Eq. (26) corresponding to different combinations of  $[S_H, S_L]$ , which are  $[(c\bar{c})_{d_6 d_3 S_H} \otimes [L \otimes Q]_{d'_6 d'_3 S_L}]$ .

	$J = 0$		$J = 1$				$J = 2$			
	[0, 0]	[1, 1]	[0, 1]	[1, 0]	[1, 1]	[1, 2]	[0, 2]	[1, 1]	[1, 2]	[1, 3]
$ \eta_c(1^1 S_0)\pi/\eta/\sigma\rangle$	1	0	—	—	—	—	—	—	—	—
$ J/\psi(1^3 S_1)\pi/\eta/\sigma\rangle$	—	—	0	1	0	0	—	—	—	—
$ h_c(1^1 P_1)\pi/\eta/\sigma\rangle$	—	—	1	0	0	0	—	—	—	—
$ \chi_{c0}(1^3 P_0)\pi/\eta/\sigma\rangle$	0	1	—	—	—	—	—	—	—	—
$ \chi_{c1}(1^3 P_1)\pi/\eta/\sigma\rangle$	—	—	0	0	1	0	—	—	—	—
$ \chi_{c2}(1^3 P_2)\pi/\eta/\sigma\rangle$	—	—	—	—	—	—	0	1	0	0
$ \eta_{c2}(1^1 D_2)\pi/\eta/\sigma\rangle$	—	—	—	—	—	—	1	0	0	0
$ \psi(1^3 D_1)\pi/\eta/\sigma\rangle$	—	—	0	0	0	1	—	—	—	—
$ \psi(1^3 D_2)\pi/\eta/\sigma\rangle$	—	—	—	—	—	—	0	0	1	0
$ \eta_c(1^1 S_0)\rho/\omega\rangle$	—	—	1	0	0	0	—	—	—	—
$ J/\psi(1^3 S_1)\rho/\omega\rangle$	0	1	0	0	1	0	0	1	0	0
$ h_c(1^1 P_1)\rho/\omega\rangle$	1	0	1	0	0	0	1	0	0	0
$ \chi_{c0}(1^3 P_0)\rho/\omega\rangle$	—	—	0	$\frac{1}{3}$	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{5}}{3}$	—	—	—	—
$ \chi_{c1}(1^3 P_1)\rho/\omega\rangle$	0	1	0	$-\frac{\sqrt{3}}{3}$	$\frac{1}{2}$	$\frac{\sqrt{15}}{6}$	0	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
$ \chi_{c2}(1^3 P_2)\rho/\omega\rangle$	—	—	0	$\frac{\sqrt{5}}{3}$	$\frac{\sqrt{15}}{6}$	$\frac{1}{6}$	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0
$ \eta_{c2}(1^1 D_2)\rho/\omega\rangle$	—	—	1	0	0	0	1	0	0	0
$ \psi(1^3 D_1)\rho/\omega\rangle$	0	1	0	0	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0	$\frac{1}{10}$	$-\frac{\sqrt{15}}{10}$	$\frac{\sqrt{21}}{5}$
$ \psi(1^3 D_2)\rho/\omega\rangle$	—	—	0	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{\sqrt{15}}{10}$	$\frac{5}{6}$	$\frac{\sqrt{35}}{15}$

There are also four tetraquark states with  $J^{PC} = 1^{++}$ . The color singlet and color octet terms of the four states contain the heavy and light spin triplet only. The states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  have the same color-spin wave functions. Their strong decay patterns are the same if we ignore their phase space difference.

There are six S-wave  $1^{+-}$  tetraquark states as shown in Eqs. (40)-(41). The states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  contain the color singlet configuration only and have the same re-coupling coefficients, while the state  $|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$  and  $|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$  have the color octet configurations only. The two states also have the same re-coupling coefficients.

### B. Decay patterns under the heavy quark symmetry

With the help of the heavy quark symmetry, we are ready to discuss the strong decay behavior of the S-wave tetraquarks. In the decay processes, the heavy spin, light spin, parity, C-parity, G-parity and isospin should be conserved. We collect some typical decay matrix elements in Table III.

All six tetraquark states with  $I^G(J^{PC}) = 1^+(1^{+-})$  can decay into  $J/\psi\pi$ ,  $\eta_c\rho$ ,  $\eta_{c2}\rho$  through S-wave and decay into  $h_c\pi$   $\{^3P_1\}$  through P-wave. The decay modes of the states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  are dominated by their color singlet, while the decay modes of the states  $|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$  and  $|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$  are governed by the color octet configurations. The six  $1^+(1^{+-})$  states decay into  $J/\psi\pi$  through the spin configuration  $(1_H \otimes 0)_1^{+-}$ . They decay into  $\eta_c\rho$  and  $\eta_{c2}\rho$  through the spin configuration  $(0_H \otimes 1)_1^{+-}$ . However, none of the six states can decay into  $\psi(1^3 D_1)\pi$  since the  $\psi(1^3 D_1)\pi$  is governed by the spin configuration  $(1_H \otimes 2)_1^{+-}$ , which does not appear in the color-spin wave functions of the six  $1^+(1^{+-})$  tetraquark states. Therefore their decay into  $\psi(1^3 D_1)\pi$  is strongly suppressed in the heavy quark limit. Similar suppression was derived for the molecular structures with  $I^G(J^{PC}) = 1^+(1^{+-})$  [67, 68].

For the isoscalar  $1^{+-}$  tetraquark states, the allowed decay modes are  $h_c\sigma$ ,  $J/\psi\eta$ ,  $\eta_c\omega$  and  $\eta_{c2}\omega$ . The decay mode  $\psi(1^3 D_1)\eta$  is suppressed due to the non-conservation of the light spin under heavy quark symmetry.

The isovector  $1^{++}$  systems have four kinds of tetraquark states. All of them can decay into  $J/\psi\rho$ ,  $\psi(1^3 D_1)\rho$  and  $\psi(1^3 D_2)\rho$  via S-wave, and decay into  $\pi\chi_{cJ}$  ( $J = 0, 1, 2$ ) via P-wave. It's interesting to note that the hidden-charm molecular states with the above quantum numbers have the same decay patterns. And the decay modes are all dominated by the spin configuration



TABLE II: The decay matrix elements of the S-wave tetraquarks under the heavy quark symmetry. The reduced matrix elements  $H_\alpha^{ij} \propto \langle Q, i | H_{eff}(\alpha) | j \rangle$ , where the indices  $i$  and  $j$  denote the light spin of the final and initial hadron respectively, and  $Q$  is the angular momentum of the final light meson. The quantum numbers in the bracelets represent the total angular momentum configurations of the final state particles.

Initial state		Final state					
$1^+(1^{++})$	$J/\psi(1^3S_1)\pi$	$\psi(1^3D_1)\pi$	$\eta_c\rho$	$\eta_{c2}\rho$	$h_c\pi \{^3P_1\}$		
$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	$\frac{\sqrt{2}}{2} H_\pi^{00}$	0	$-\frac{\sqrt{2}}{2} H_\rho^{01}$	$-\frac{\sqrt{2}}{2} H_\rho^{21}$	$-\frac{\sqrt{2}}{2} H_\pi^{11}$		
$ 35_{cs}, 1_c, 1\rangle_{(15,15)}$	$\frac{\sqrt{2}}{2} H_\pi^{00}$	0	$-\frac{\sqrt{2}}{2} H_\rho^{01}$	$-\frac{\sqrt{2}}{2} H_\rho^{21}$	$-\frac{\sqrt{2}}{2} H_\pi^{11}$		
$ 35_{cs}, 1_c, 1\rangle_{(21,15)}$	$\frac{\sqrt{3}}{3} H_\pi^{00} + \frac{\sqrt{6}}{6} \tilde{H}_\pi^{00}$	0	$\frac{\sqrt{3}}{3} H_\rho^{01} + \frac{\sqrt{6}}{6} \tilde{H}_\rho^{01}$	$\frac{\sqrt{3}}{3} H_\rho^{21} + \frac{\sqrt{6}}{6} \tilde{H}_\rho^{21}$	$\frac{\sqrt{3}}{3} H_\pi^{11} + \frac{\sqrt{6}}{6} \tilde{H}_\pi^{11}$		
$ 35_{cs}, 1_c, 1\rangle_{(15,21)}$	$\frac{\sqrt{6}}{6} H_\pi^{00} - \frac{\sqrt{3}}{3} \tilde{H}_\pi^{00}$	0	$\frac{\sqrt{6}}{6} H_\rho^{01} - \frac{\sqrt{3}}{3} \tilde{H}_\rho^{01}$	$\frac{\sqrt{6}}{6} H_\rho^{21} - \frac{\sqrt{3}}{3} \tilde{H}_\rho^{21}$	$\frac{\sqrt{6}}{6} H_\pi^{11} - \frac{\sqrt{3}}{3} \tilde{H}_\pi^{11}$		
$ 280_{cs}, 1_c, 1\rangle_{(21,15)}$	$-\frac{\sqrt{2}}{2} \tilde{H}_\pi^{00}$	0	$\frac{\sqrt{2}}{2} \tilde{H}_\rho^{01}$	$\frac{\sqrt{2}}{2} \tilde{H}_\rho^{21}$	$\frac{\sqrt{2}}{2} \tilde{H}_\pi^{11}$		
$ 280_{cs}, 1_c, 1\rangle_{(15,21)}$	$-\frac{\sqrt{2}}{2} \tilde{H}_\pi^{00}$	0	$\frac{\sqrt{2}}{2} \tilde{H}_\rho^{01}$	$\frac{\sqrt{2}}{2} \tilde{H}_\rho^{21}$	$\frac{\sqrt{2}}{2} \tilde{H}_\pi^{11}$		
Initial state		Final state					
$1^-(1^{++})$	$J/\psi(1^3S_1)\rho$	$\psi(1^3D_1)\rho$	$\psi(1^3D_2)\rho$	$\chi_{c0}\pi \{^3P_1\}$	$\chi_{c1}\pi \{^3P_1\}$	$\chi_{c2}\pi \{^3P_1\}$	
$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	$-\frac{1}{3} H_\rho^{01} - \frac{2\sqrt{2}}{3} \tilde{H}_\rho^{01}$	$\frac{1}{6} H_\rho^{21} + \frac{\sqrt{2}}{3} \tilde{H}_\rho^{21}$	$-\frac{\sqrt{3}}{6} H_\rho^{21} - \frac{\sqrt{6}}{6} \tilde{H}_\rho^{21}$	$\frac{\sqrt{3}}{9} H_\pi^{11} + \frac{2}{3} \tilde{H}_\pi^{11}$	$-\frac{1}{6} H_\pi^{11} - \frac{\sqrt{2}}{3} \tilde{H}_\pi^{11}$	$-\frac{\sqrt{15}}{18} H_\pi^{11} - \frac{\sqrt{5}}{3} \tilde{H}_\pi^{11}$	
$ 35_{cs}, 1_c, 1\rangle_{(15,15)}$	$-\frac{1}{3} H_\rho^{01} - \frac{2\sqrt{2}}{3} \tilde{H}_\rho^{01}$	$\frac{1}{6} H_\rho^{21} + \frac{\sqrt{2}}{3} \tilde{H}_\rho^{21}$	$-\frac{\sqrt{3}}{6} H_\rho^{21} - \frac{\sqrt{6}}{6} \tilde{H}_\rho^{21}$	$\frac{\sqrt{3}}{9} H_\pi^{11} + \frac{2}{3} \tilde{H}_\pi^{11}$	$-\frac{1}{6} H_\pi^{11} - \frac{\sqrt{2}}{3} \tilde{H}_\pi^{11}$	$-\frac{\sqrt{15}}{18} H_\pi^{11} - \frac{\sqrt{5}}{3} \tilde{H}_\pi^{11}$	
$ 280_{cs}, 1_c, 1\rangle_{(21,15)}$	$\frac{2\sqrt{2}}{3} H_\rho^{01} - \frac{1}{3} \tilde{H}_\rho^{01}$	$-\frac{\sqrt{2}}{3} H_\rho^{21} + \frac{1}{6} \tilde{H}_\rho^{21}$	$\frac{\sqrt{6}}{6} H_\rho^{21} - \frac{\sqrt{3}}{6} \tilde{H}_\rho^{21}$	$-\frac{2}{3} H_\pi^{11} + \frac{\sqrt{2}}{9} \tilde{H}_\pi^{11}$	$\frac{\sqrt{2}}{3} H_\pi^{11} - \frac{1}{6} \tilde{H}_\pi^{11}$	$\frac{\sqrt{5}}{3} H_\pi^{11} - \frac{\sqrt{15}}{18} \tilde{H}_\pi^{11}$	
$ 280_{cs}, 1_c, 1\rangle_{(15,21)}$	$\frac{2\sqrt{2}}{3} H_\rho^{01} - \frac{1}{3} \tilde{H}_\rho^{01}$	$-\frac{\sqrt{2}}{3} H_\rho^{21} + \frac{1}{6} \tilde{H}_\rho^{21}$	$\frac{\sqrt{6}}{6} H_\rho^{21} - \frac{\sqrt{3}}{6} \tilde{H}_\rho^{21}$	$-\frac{2}{3} H_\pi^{11} + \frac{\sqrt{2}}{9} \tilde{H}_\pi^{11}$	$\frac{\sqrt{2}}{3} H_\pi^{11} - \frac{1}{6} \tilde{H}_\pi^{11}$	$\frac{\sqrt{5}}{3} H_\pi^{11} - \frac{\sqrt{15}}{18} \tilde{H}_\pi^{11}$	
Initial state		Final state					
$0^-(1^{++})$	$h_c\sigma$	$J/\psi(1^3S_1)\eta$	$\psi(1^3D_1)\eta$	$\eta_c\omega$	$\eta_{c2}\omega$		
$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	$-\frac{\sqrt{2}}{2} H_\sigma^{11}$	$\frac{\sqrt{2}}{2} H_\eta^{00}$	0	$-\frac{\sqrt{2}}{2} H_\omega^{01}$	$-\frac{\sqrt{2}}{2} H_\omega^{21}$		
$ 35_{cs}, 1_c, 1\rangle_{(15,15)}$	$-\frac{\sqrt{2}}{2} H_\sigma^{11}$	$\frac{\sqrt{2}}{2} H_\eta^{00}$	0	$-\frac{\sqrt{2}}{2} H_\omega^{01}$	$-\frac{\sqrt{2}}{2} H_\omega^{21}$		
$ 35_{cs}, 1_c, 1\rangle_{(21,15)}$	$\frac{\sqrt{3}}{3} H_\sigma^{11} + \frac{\sqrt{6}}{6} \tilde{H}_\sigma^{11}$	$\frac{\sqrt{3}}{3} H_\eta^{00} + \frac{\sqrt{6}}{6} \tilde{H}_\eta^{00}$	0	$\frac{\sqrt{3}}{3} H_\omega^{01} + \frac{\sqrt{6}}{6} \tilde{H}_\omega^{01}$	$\frac{\sqrt{3}}{3} H_\omega^{21} + \frac{\sqrt{6}}{6} \tilde{H}_\omega^{21}$		
$ 35_{cs}, 1_c, 1\rangle_{(15,21)}$	$\frac{\sqrt{6}}{6} H_\sigma^{11} - \frac{\sqrt{3}}{3} \tilde{H}_\sigma^{11}$	$\frac{\sqrt{6}}{6} H_\eta^{00} - \frac{\sqrt{3}}{3} \tilde{H}_\eta^{00}$	0	$\frac{\sqrt{6}}{6} H_\omega^{01} - \frac{\sqrt{3}}{3} \tilde{H}_\omega^{01}$	$\frac{\sqrt{6}}{6} H_\omega^{21} - \frac{\sqrt{3}}{3} \tilde{H}_\omega^{21}$		
$ 280_{cs}, 1_c, 1\rangle_{(21,15)}$	$\frac{\sqrt{2}}{2} \tilde{H}_\sigma^{11}$	$-\frac{\sqrt{2}}{2} \tilde{H}_\eta^{00}$	0	$\frac{\sqrt{2}}{2} \tilde{H}_\omega^{01}$	$\frac{\sqrt{2}}{2} \tilde{H}_\omega^{21}$		
$ 280_{cs}, 1_c, 1\rangle_{(15,21)}$	$\frac{\sqrt{2}}{2} \tilde{H}_\sigma^{11}$	$-\frac{\sqrt{2}}{2} \tilde{H}_\eta^{00}$	0	$\frac{\sqrt{2}}{2} \tilde{H}_\omega^{01}$	$\frac{\sqrt{2}}{2} \tilde{H}_\omega^{21}$		
Initial state		Final state					
$0^+(1^{++})$	$\chi_{c1}\sigma$	$J/\psi(1^3S_1)\omega$	$\psi(1^3D_1)\omega$	$\psi(1^3D_2)\omega$	$\chi_{c0}\eta \{^3P_1\}$	$\chi_{c1}\eta \{^3P_1\}$	$\chi_{c2}\eta \{^3P_1\}$
$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	$-\frac{1}{3} H_\sigma^{01} - \frac{2\sqrt{2}}{3} \tilde{H}_\sigma^{01}$	$-\frac{1}{3} H_\omega^{01} - \frac{2\sqrt{2}}{3} \tilde{H}_\omega^{01}$	$\frac{1}{6} H_\omega^{21} + \frac{\sqrt{2}}{3} \tilde{H}_\omega^{21}$	$-\frac{\sqrt{3}}{6} H_\omega^{21} - \frac{\sqrt{6}}{6} \tilde{H}_\omega^{21}$	$\frac{\sqrt{3}}{9} H_\eta^{11} + \frac{2}{3} \tilde{H}_\eta^{11}$	$-\frac{1}{6} H_\eta^{11} - \frac{\sqrt{2}}{3} \tilde{H}_\eta^{11}$	$-\frac{\sqrt{15}}{18} H_\eta^{11} - \frac{\sqrt{5}}{3} \tilde{H}_\eta^{11}$
$ 35_{cs}, 1_c, 1\rangle_{(15,15)}$	$-\frac{1}{3} H_\sigma^{01} - \frac{2\sqrt{2}}{3} \tilde{H}_\sigma^{01}$	$-\frac{1}{3} H_\omega^{01} - \frac{2\sqrt{2}}{3} \tilde{H}_\omega^{01}$	$\frac{1}{6} H_\omega^{21} + \frac{\sqrt{2}}{3} \tilde{H}_\omega^{21}$	$-\frac{\sqrt{3}}{6} H_\omega^{21} - \frac{\sqrt{6}}{6} \tilde{H}_\omega^{21}$	$\frac{\sqrt{3}}{9} H_\eta^{11} + \frac{2}{3} \tilde{H}_\eta^{11}$	$-\frac{1}{6} H_\eta^{11} - \frac{\sqrt{2}}{3} \tilde{H}_\eta^{11}$	$-\frac{\sqrt{15}}{18} H_\eta^{11} - \frac{\sqrt{5}}{3} \tilde{H}_\eta^{11}$
$ 280_{cs}, 1_c, 1\rangle_{(21,15)}$	$\frac{2\sqrt{2}}{3} H_\sigma^{01} - \frac{1}{3} \tilde{H}_\sigma^{01}$	$\frac{2\sqrt{2}}{3} H_\omega^{01} - \frac{1}{3} \tilde{H}_\omega^{01}$	$-\frac{\sqrt{2}}{3} H_\omega^{21} + \frac{1}{6} \tilde{H}_\omega^{21}$	$\frac{\sqrt{6}}{6} H_\omega^{21} - \frac{\sqrt{3}}{6} \tilde{H}_\omega^{21}$	$-\frac{2}{3} H_\eta^{11} + \frac{\sqrt{2}}{9} \tilde{H}_\eta^{11}$	$\frac{\sqrt{2}}{3} H_\eta^{11} - \frac{1}{6} \tilde{H}_\eta^{11}$	$\frac{\sqrt{5}}{3} H_\eta^{11} - \frac{\sqrt{15}}{18} \tilde{H}_\eta^{11}$
$ 280_{cs}, 1_c, 1\rangle_{(15,21)}$	$\frac{2\sqrt{2}}{3} H_\sigma^{01} - \frac{1}{3} \tilde{H}_\sigma^{01}$	$\frac{2\sqrt{2}}{3} H_\omega^{01} - \frac{1}{3} \tilde{H}_\omega^{01}$	$-\frac{\sqrt{2}}{3} H_\omega^{21} + \frac{1}{6} \tilde{H}_\omega^{21}$	$\frac{\sqrt{6}}{6} H_\omega^{21} - \frac{\sqrt{3}}{6} \tilde{H}_\omega^{21}$	$-\frac{2}{3} H_\eta^{11} + \frac{\sqrt{2}}{9} \tilde{H}_\eta^{11}$	$\frac{\sqrt{2}}{3} H_\eta^{11} - \frac{1}{6} \tilde{H}_\eta^{11}$	$\frac{\sqrt{5}}{3} H_\eta^{11} - \frac{\sqrt{15}}{18} \tilde{H}_\eta^{11}$
Initial state		Final state					
$1^-(0^{++})$	$\eta_c\pi$	$J/\psi(1^3S_1)\rho$	$\psi(1^3D_1)\rho$	$\chi_{c1}\pi \{^3P_0\}$			
$ 1_{cs}, 1_c, 0\rangle_{(21,21)}$	$\frac{\sqrt{21}}{6} H_\pi^{00} + \frac{\sqrt{42}}{21} \tilde{H}_\pi^{00}$	$-\frac{\sqrt{7}}{14} H_\rho^{01} - \frac{\sqrt{14}}{14} \tilde{H}_\rho^{01}$	$-\frac{\sqrt{7}}{14} H_\rho^{21} - \frac{\sqrt{14}}{14} \tilde{H}_\rho^{21}$	$-\frac{\sqrt{7}}{14} H_\pi^{11} - \frac{\sqrt{14}}{14} \tilde{H}_\pi^{11}$			
$ 405_{cs}, 1_c, 0\rangle_{(21,21)}$	$\frac{3\sqrt{7}}{14} \tilde{H}_\pi^{00}$	$-\frac{2\sqrt{42}}{21} H_\rho^{01} + \frac{5\sqrt{21}}{42} \tilde{H}_\rho^{01}$	$-\frac{2\sqrt{42}}{21} H_\rho^{21} + \frac{5\sqrt{21}}{42} \tilde{H}_\rho^{21}$	$-\frac{2\sqrt{42}}{21} H_\pi^{11} + \frac{5\sqrt{21}}{42} \tilde{H}_\pi^{11}$			
$ 1_{cs}, 1_c, 0\rangle_{(15,15)}$	$\frac{\sqrt{15}}{6} H_\pi^{00} - \frac{\sqrt{30}}{15} \tilde{H}_\pi^{00}$	$\frac{\sqrt{5}}{10} H_\rho^{01} + \frac{\sqrt{10}}{10} \tilde{H}_\rho^{01}$	$\frac{\sqrt{5}}{10} H_\rho^{21} + \frac{\sqrt{10}}{10} \tilde{H}_\rho^{21}$	$\frac{\sqrt{5}}{10} H_\pi^{11} + \frac{\sqrt{10}}{10} \tilde{H}_\pi^{11}$			
$ 189_{cs}, 1_c, 0\rangle_{(15,15)}$	$-\frac{3\sqrt{5}}{10} \tilde{H}_\pi^{00}$	$-\frac{2\sqrt{30}}{15} H_\rho^{01} - \frac{\sqrt{15}}{30} \tilde{H}_\rho^{01}$	$-\frac{2\sqrt{30}}{15} H_\rho^{21} - \frac{\sqrt{15}}{30} \tilde{H}_\rho^{21}$	$-\frac{2\sqrt{30}}{15} H_\pi^{11} - \frac{\sqrt{15}}{30} \tilde{H}_\pi^{11}$			
Initial state		Final state					
$0^+(0^{++})$	$\chi_{c0}\sigma$	$\eta_c\eta$	$J/\psi(1^3S_1)\omega$	$\psi(1^3D_1)\omega$	$\chi_{c1}\eta \{^3P_0\}$		
$ 1_{cs}, 1_c, 0\rangle_{(21,21)}$	$-\frac{\sqrt{7}}{14} H_\sigma^{01} - \frac{\sqrt{14}}{14} \tilde{H}_\sigma^{01}$	$\frac{\sqrt{21}}{6} H_\eta^{00} + \frac{\sqrt{42}}{21} \tilde{H}_\eta^{00}$	$-\frac{\sqrt{7}}{14} H_\omega^{01} - \frac{\sqrt{14}}{14} \tilde{H}_\omega^{01}$	$-\frac{\sqrt{7}}{14} H_\omega^{21} - \frac{\sqrt{14}}{14} \tilde{H}_\omega^{21}$	$-\frac{\sqrt{7}}{14} H_\eta^{11} - \frac{\sqrt{14}}{14} \tilde{H}_\eta^{11}$		
$ 405_{cs}, 1_c, 0\rangle_{(21,21)}$	$-\frac{2\sqrt{42}}{21} H_\sigma^{01} + \frac{5\sqrt{21}}{42} \tilde{H}_\sigma^{01}$	$\frac{3\sqrt{7}}{14} \tilde{H}_\eta^{00}$	$-\frac{2\sqrt{42}}{21} H_\omega^{01} + \frac{5\sqrt{21}}{42} \tilde{H}_\omega^{01}$	$-\frac{2\sqrt{42}}{21} H_\omega^{21} + \frac{5\sqrt{21}}{42} \tilde{H}_\omega^{21}$	$-\frac{2\sqrt{42}}{21} H_\eta^{11} + \frac{5\sqrt{21}}{42} \tilde{H}_\eta^{11}$		
$ 1_{cs}, 1_c, 0\rangle_{(15,15)}$	$\frac{\sqrt{5}}{10} H_\sigma^{01} + \frac{\sqrt{10}}{10} \tilde{H}_\sigma^{01}$	$\frac{\sqrt{15}}{6} H_\eta^{00} - \frac{\sqrt{30}}{15} \tilde{H}_\eta^{00}$	$\frac{\sqrt{5}}{10} H_\omega^{01} + \frac{\sqrt{10}}{10} \tilde{H}_\omega^{01}$	$\frac{\sqrt{5}}{10} H_\omega^{21} + \frac{\sqrt{10}}{10} \tilde{H}_\omega^{21}$	$\frac{\sqrt{5}}{10} H_\eta^{11} + \frac{\sqrt{10}}{10} \tilde{H}_\eta^{11}$		
$ 189_{cs}, 1_c, 0\rangle_{(15,15)}$	$-\frac{2\sqrt{30}}{15} H_\sigma^{01} - \frac{\sqrt{15}}{30} \tilde{H}_\sigma^{01}$	$-\frac{3\sqrt{5}}{10} \tilde{H}_\eta^{00}$	$-\frac{2\sqrt{30}}{15} H_\omega^{01} - \frac{\sqrt{15}}{30} \tilde{H}_\omega^{01}$	$-\frac{2\sqrt{30}}{15} H_\omega^{21} - \frac{\sqrt{15}}{30} \tilde{H}_\omega^{21}$	$-\frac{2\sqrt{30}}{15} H_\eta^{11} - \frac{\sqrt{15}}{30} \tilde{H}_\eta^{11}$		

$(1_H \otimes 1_l)^{++}$ .

The  $0^+(1^{++})$  tetraquark states have the S-wave decay modes such as  $\chi_{c1}\sigma$ ,  $J/\psi\omega$ ,  $\psi(1^3D_1)\omega$  and  $\psi(1^3D_2)\omega$ . They also have the P-wave decay modes  $\chi_{c1}\eta$  ( $J = 0, 1, 2$ ). All the decays occur through the spin configuration  $(1_H \otimes 1_l)_0^{++}$ . The color octet configurations also contribute to the decay.

The allowed decay modes of the  $1^-(0^{++})$  tetraquark states are  $\eta_c\pi$ ,  $J/\psi\rho$  and  $\psi(1^3D_1)\rho$ . The P-wave decay mode  $\chi_{c1}\pi$  is also allowed in the heavy quark limit. The decay mode  $\eta_c\pi$  is governed by the spin configuration  $(0_H \otimes 0_l)_0^{++}$ . The S-wave decay modes  $J/\psi\rho$ ,  $\psi(1^3D_1)\rho$  and the P-wave decay mode  $\chi_{c1}\pi$  are all dominated by the spin configuration  $(1_H \otimes 1_l)_0^{++}$ . As listed in Table III, the decay mode  $\eta_c\pi$  for the states  $|1_{cs}, 1_c, 0\rangle_{(21,21)}$  and  $|1_{cs}, 1_c, 0\rangle_{(15,15)}$  arises from both the color singlet and color octet configurations. For the states  $|405_{cs}, 1_c, 0\rangle_{(21,21)}$  and  $|189_{cs}, 1_c, 0\rangle_{(15,15)}$ , only the color singlet contributes to the decay.

TABLE III: The decay matrix elements of the S-wave tetraquarks with the constraint of the color-spin  $SU(6)_{cs}$  symmetry. The reduced matrix elements  $H_{\alpha}^{ij} \propto \langle Q, i || H_{eff}(\alpha) || j \rangle$ , where the indices  $i$  and  $j$  denote the light spin of the final and initial hadron respectively, and  $Q$  is the angular momentum of the final light meson. The quantum numbers in the bracelets represent the total angular momentum configurations of the final state particles.

$I^G(J^{PC})$	Initial state	Final state				
		$J/\psi(1^3S_1)\pi$	$\psi(1^3D_1)\pi$	$\eta_c\rho$	$\eta_{c2}\rho$	$h_c\pi\{^3P_1\}$
$1^+(1^{+-})$	$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	$\frac{\sqrt{2}}{2}H_{\pi}^{00}$	0	$-\frac{\sqrt{2}}{2}H_{\rho}^{01}$	$-\frac{\sqrt{2}}{2}H_{\rho}^{21}$	$\epsilon$
	$ 35_{cs}, 1_c, 1\rangle_{(15,15)}$	$\frac{\sqrt{2}}{2}H_{\pi}^{00}$	0	$-\frac{\sqrt{2}}{2}H_{\rho}^{01}$	$-\frac{\sqrt{2}}{2}H_{\rho}^{21}$	$\epsilon$
	$ 35_{cs}, 1_c, 1\rangle_{(21,15)}$	$\frac{\sqrt{3}}{3}H_{\pi}^{00} + \tilde{\epsilon}$	0	$\frac{\sqrt{3}}{3}H_{\rho}^{01} + \tilde{\epsilon}$	$\frac{\sqrt{3}}{3}H_{\rho}^{21} + \tilde{\epsilon}$	$\epsilon + \tilde{\epsilon}$
	$ 35_{cs}, 1_c, 1\rangle_{(15,21)}$	$\frac{\sqrt{6}}{6}H_{\pi}^{00} - \tilde{\epsilon}$	0	$\frac{\sqrt{6}}{6}H_{\rho}^{01} - \tilde{\epsilon}$	$\frac{\sqrt{6}}{6}H_{\rho}^{21} - \tilde{\epsilon}$	$\epsilon - \tilde{\epsilon}$
	$ 280_{cs}, 1_c, 1\rangle_{(21,15)}$	$\tilde{\epsilon}$	0	$\tilde{\epsilon}$	$\tilde{\epsilon}$	$\tilde{\epsilon}$
	$ 280_{cs}, 1_c, 1\rangle_{(15,21)}$	$\tilde{\epsilon}$	0	$\tilde{\epsilon}$	$\tilde{\epsilon}$	$\tilde{\epsilon}$
$I^G(J^{PC})$	Initial state	Final state				
		$J/\psi(1^3S_1)\rho$	$\psi(1^3D_1)\rho$	$\psi(1^3D_2)\rho$	$\chi_{c0}\pi\{^3P_1\}$	$\chi_{c1}\pi\{^3P_1\}$ $\chi_{c2}\pi\{^3P_1\}$
$1^-(1^{++})$	$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	$-\frac{1}{3}H_{\rho}^{01} - \frac{2\sqrt{2}}{3}\tilde{H}_{\rho}^{01}$	$\frac{1}{6}H_{\rho}^{21} + \frac{\sqrt{2}}{3}\tilde{H}_{\rho}^{21}$	$-\frac{\sqrt{3}}{6}H_{\rho}^{21} - \frac{\sqrt{6}}{3}\tilde{H}_{\rho}^{21}$	$\epsilon + \tilde{\epsilon}$	$-\epsilon - \tilde{\epsilon}$ $-\epsilon - \tilde{\epsilon}$
	$ 35_{cs}, 1_c, 1\rangle_{(15,15)}$	$-\frac{1}{3}H_{\rho}^{01} - \frac{2\sqrt{2}}{3}\tilde{H}_{\rho}^{01}$	$\frac{1}{6}H_{\rho}^{21} + \frac{\sqrt{2}}{3}\tilde{H}_{\rho}^{21}$	$-\frac{\sqrt{3}}{6}H_{\rho}^{21} - \frac{\sqrt{6}}{3}\tilde{H}_{\rho}^{21}$	$\epsilon + \tilde{\epsilon}$	$-\epsilon - \tilde{\epsilon}$ $-\epsilon - \tilde{\epsilon}$
	$ 280_{cs}, 1_c, 1\rangle_{(21,15)}$	$\frac{2\sqrt{2}}{3}H_{\rho}^{01} - \frac{1}{3}\tilde{H}_{\rho}^{01}$	$-\frac{\sqrt{2}}{3}H_{\rho}^{21} + \frac{1}{6}\tilde{H}_{\rho}^{21}$	$\frac{\sqrt{6}}{3}H_{\rho}^{21} - \frac{\sqrt{3}}{6}\tilde{H}_{\rho}^{21}$	$-\epsilon + \tilde{\epsilon}$	$\epsilon - \tilde{\epsilon}$ $\epsilon - \tilde{\epsilon}$
	$ 280_{cs}, 1_c, 1\rangle_{(15,21)}$	$\frac{2\sqrt{2}}{3}H_{\rho}^{01} - \frac{1}{3}\tilde{H}_{\rho}^{01}$	$-\frac{\sqrt{2}}{3}H_{\rho}^{21} + \frac{1}{6}\tilde{H}_{\rho}^{21}$	$\frac{\sqrt{6}}{3}H_{\rho}^{21} - \frac{\sqrt{3}}{6}\tilde{H}_{\rho}^{21}$	$-\epsilon + \tilde{\epsilon}$	$\epsilon - \tilde{\epsilon}$ $\epsilon - \tilde{\epsilon}$
$I^G(J^{PC})$	Initial state	Final state				
		$h_c\sigma$	$J/\psi(1^3S_1)\eta$	$\psi(1^3D_1)\eta$	$\eta_c\omega$	$\eta_{c2}\omega$
$0^-(1^{+-})$	$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	$-\epsilon$	$\frac{\sqrt{2}}{2}H_{\eta}^{00}$	0	$-\frac{\sqrt{2}}{2}H_{\omega}^{01}$	$-\frac{\sqrt{2}}{2}H_{\omega}^{21}$
	$ 35_{cs}, 1_c, 1\rangle_{(15,15)}$	$-\epsilon$	$\frac{\sqrt{2}}{2}H_{\eta}^{00}$	0	$-\frac{\sqrt{2}}{2}H_{\omega}^{01}$	$-\frac{\sqrt{2}}{2}H_{\omega}^{21}$
	$ 35_{cs}, 1_c, 1\rangle_{(21,15)}$	$\epsilon + \tilde{\epsilon}$	$\frac{\sqrt{3}}{3}H_{\eta}^{00} + \tilde{\epsilon}$	0	$\frac{\sqrt{3}}{3}H_{\omega}^{01} + \tilde{\epsilon}$	$\frac{\sqrt{3}}{3}H_{\omega}^{21} + \tilde{\epsilon}$
	$ 35_{cs}, 1_c, 1\rangle_{(15,21)}$	$\epsilon - \tilde{\epsilon}$	$\frac{\sqrt{6}}{6}H_{\eta}^{00} - \tilde{\epsilon}$	0	$\frac{\sqrt{6}}{6}H_{\omega}^{01} - \tilde{\epsilon}$	$\frac{\sqrt{6}}{6}H_{\omega}^{21} - \tilde{\epsilon}$
	$ 280_{cs}, 1_c, 1\rangle_{(21,15)}$	$\tilde{\epsilon}$	$-\tilde{\epsilon}$	0	$\tilde{\epsilon}$	$\tilde{\epsilon}$
	$ 280_{cs}, 1_c, 1\rangle_{(15,21)}$	$\tilde{\epsilon}$	$-\tilde{\epsilon}$	0	$\tilde{\epsilon}$	$\tilde{\epsilon}$
$I^G(J^{PC})$	Initial state	Final state				
		$\chi_{c1}\sigma$	$J/\psi(1^3S_1)\omega$	$\psi(1^3D_1)\omega$	$\psi(1^3D_2)\omega$	$\chi_{c0}\eta\{^3P_1\}$ $\chi_{c1}\eta\{^3P_1\}$ $\chi_{c2}\eta\{^3P_1\}$
$0^+(1^{++})$	$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	$-\epsilon - \tilde{\epsilon}$	$-\frac{1}{3}H_{\omega}^{01} - \frac{2\sqrt{2}}{3}\tilde{H}_{\omega}^{01}$	$\frac{1}{6}H_{\omega}^{21} + \frac{\sqrt{2}}{3}\tilde{H}_{\omega}^{21}$	$-\frac{\sqrt{3}}{6}H_{\omega}^{21} - \frac{\sqrt{6}}{3}\tilde{H}_{\omega}^{21}$	$\epsilon + \tilde{\epsilon}$ $-\epsilon - \tilde{\epsilon}$ $-\epsilon - \tilde{\epsilon}$
	$ 35_{cs}, 1_c, 1\rangle_{(15,15)}$	$-\epsilon - \tilde{\epsilon}$	$-\frac{1}{3}H_{\omega}^{01} - \frac{2\sqrt{2}}{3}\tilde{H}_{\omega}^{01}$	$\frac{1}{6}H_{\omega}^{21} + \frac{\sqrt{2}}{3}\tilde{H}_{\omega}^{21}$	$-\frac{\sqrt{3}}{6}H_{\omega}^{21} - \frac{\sqrt{6}}{3}\tilde{H}_{\omega}^{21}$	$\epsilon + \tilde{\epsilon}$ $-\epsilon - \tilde{\epsilon}$ $-\epsilon - \tilde{\epsilon}$
	$ 280_{cs}, 1_c, 1\rangle_{(21,15)}$	$\epsilon - \tilde{\epsilon}$	$\frac{2\sqrt{2}}{3}H_{\omega}^{01} - \frac{1}{3}\tilde{H}_{\omega}^{01}$	$-\frac{\sqrt{2}}{3}H_{\omega}^{21} + \frac{1}{6}\tilde{H}_{\omega}^{21}$	$\frac{\sqrt{6}}{3}H_{\omega}^{21} - \frac{\sqrt{3}}{6}\tilde{H}_{\omega}^{21}$	$-\epsilon + \tilde{\epsilon}$ $\epsilon - \tilde{\epsilon}$ $\epsilon - \tilde{\epsilon}$
	$ 280_{cs}, 1_c, 1\rangle_{(15,21)}$	$\epsilon - \tilde{\epsilon}$	$\frac{2\sqrt{2}}{3}H_{\omega}^{01} - \frac{1}{3}\tilde{H}_{\omega}^{01}$	$-\frac{\sqrt{2}}{3}H_{\omega}^{21} + \frac{1}{6}\tilde{H}_{\omega}^{21}$	$\frac{\sqrt{6}}{3}H_{\omega}^{21} - \frac{\sqrt{3}}{6}\tilde{H}_{\omega}^{21}$	$-\epsilon + \tilde{\epsilon}$ $\epsilon - \tilde{\epsilon}$ $\epsilon - \tilde{\epsilon}$
$I^G(J^{PC})$	Initial state	Final state				
		$\eta_c\pi$	$J/\psi(1^3S_1)\rho$	$\psi(1^3D_1)\rho$	$\chi_{c1}\pi\{^3P_0\}$	
$1^-(0^{++})$	$ 1_{cs}, 1_c, 0\rangle_{(21,21)}$	$\frac{\sqrt{21}}{6}H_{\pi}^{00} + \tilde{\epsilon}$	$-\frac{\sqrt{7}}{14}H_{\rho}^{01} - \frac{\sqrt{14}}{7}\tilde{H}_{\rho}^{01}$	$-\frac{\sqrt{7}}{14}H_{\rho}^{21} - \frac{\sqrt{14}}{7}\tilde{H}_{\rho}^{21}$	$-\epsilon - \tilde{\epsilon}$	
	$ 405_{cs}, 1_c, 0\rangle_{(21,21)}$	$\tilde{\epsilon}$	$-\frac{2\sqrt{42}}{21}H_{\rho}^{01} + \frac{5\sqrt{21}}{42}\tilde{H}_{\rho}^{01}$	$-\frac{2\sqrt{42}}{21}H_{\rho}^{21} + \frac{5\sqrt{21}}{42}\tilde{H}_{\rho}^{21}$	$-\epsilon + \tilde{\epsilon}$	
	$ 1_{cs}, 1_c, 0\rangle_{(15,15)}$	$\frac{\sqrt{15}}{6}H_{\pi}^{00} - \tilde{\epsilon}$	$\frac{\sqrt{5}}{10}H_{\rho}^{01} + \frac{\sqrt{10}}{5}\tilde{H}_{\rho}^{01}$	$\frac{\sqrt{5}}{10}H_{\rho}^{21} + \frac{\sqrt{10}}{5}\tilde{H}_{\rho}^{21}$	$\epsilon + \tilde{\epsilon}$	
	$ 189_{cs}, 1_c, 0\rangle_{(15,15)}$	$-\tilde{\epsilon}$	$-\frac{2\sqrt{30}}{15}H_{\rho}^{01} - \frac{\sqrt{15}}{30}\tilde{H}_{\rho}^{01}$	$-\frac{2\sqrt{30}}{15}H_{\rho}^{21} - \frac{\sqrt{15}}{30}\tilde{H}_{\rho}^{21}$	$-\epsilon - \tilde{\epsilon}$	
$I^G(J^{PC})$	Initial state	Final state				
		$\chi_{c0}\sigma$	$\eta_c\eta$	$J/\psi(1^3S_1)\omega$	$\psi(1^3D_1)\omega$	$\chi_{c1}\eta\{^3P_0\}$
$0^+(0^{++})$	$ 1_{cs}, 1_c, 0\rangle_{(21,21)}$	$-\epsilon - \tilde{\epsilon}$	$\frac{\sqrt{21}}{6}H_{\eta}^{00} + \tilde{\epsilon}$	$-\frac{\sqrt{7}}{14}H_{\omega}^{01} - \frac{\sqrt{14}}{7}\tilde{H}_{\omega}^{01}$	$-\frac{\sqrt{7}}{14}H_{\omega}^{21} - \frac{\sqrt{14}}{7}\tilde{H}_{\omega}^{21}$	$-\epsilon - \tilde{\epsilon}$
	$ 405_{cs}, 1_c, 0\rangle_{(21,21)}$	$-\epsilon + \tilde{\epsilon}$	$\tilde{\epsilon}$	$-\frac{2\sqrt{42}}{21}H_{\omega}^{01} + \frac{5\sqrt{21}}{42}\tilde{H}_{\omega}^{01}$	$-\frac{2\sqrt{42}}{21}H_{\omega}^{21} + \frac{5\sqrt{21}}{42}\tilde{H}_{\omega}^{21}$	$-\epsilon + \tilde{\epsilon}$
	$ 1_{cs}, 1_c, 0\rangle_{(15,15)}$	$\epsilon + \tilde{\epsilon}$	$\frac{\sqrt{15}}{6}H_{\eta}^{00} - \tilde{\epsilon}$	$\frac{\sqrt{5}}{10}H_{\omega}^{01} + \frac{\sqrt{10}}{5}\tilde{H}_{\omega}^{01}$	$\frac{\sqrt{5}}{10}H_{\omega}^{21} + \frac{\sqrt{10}}{5}\tilde{H}_{\omega}^{21}$	$\epsilon + \tilde{\epsilon}$
	$ 189_{cs}, 1_c, 0\rangle_{(15,15)}$	$-\epsilon - \tilde{\epsilon}$	$-\tilde{\epsilon}$	$-\frac{2\sqrt{30}}{15}H_{\omega}^{01} - \frac{\sqrt{15}}{30}\tilde{H}_{\omega}^{01}$	$-\frac{2\sqrt{30}}{15}H_{\omega}^{21} - \frac{\sqrt{15}}{30}\tilde{H}_{\omega}^{21}$	$-\epsilon - \tilde{\epsilon}$

The isoscalar  $0^{++}$  tetraquark states have the S-wave decay modes  $\chi_{c0}\sigma$ ,  $\eta_c\eta$ ,  $J/\psi\omega$ ,  $\psi(1^3D_1)\omega$  and the P-wave decay mode  $\chi_{c1}\eta$ . Similar to their isovector partners, the decay mode  $\eta_c\eta$  for the states  $|1_{cs}, 1_c, 0\rangle_{(21,21)}$  and  $|1_{cs}, 1_c, 0\rangle_{(15,15)}$  are from the contributions of both the color singlet and color octet configurations. The states  $|405_{cs}, 1_c, 0\rangle_{(21,21)}$  and  $|189_{cs}, 1_c, 0\rangle_{(15,15)}$  decay via the color singlet only.

The above discussions are based on the heavy quark symmetry without considering any decay dynamics. In fact, the decay patterns shown in Table III are the same as the decay patterns within the molecule framework listed in Refs. [67, 68] for a hidden-charm system with  $I^G(J^{PC}) = 1^+(1^{+-})$ .

### C. Further analysis of the decay patterns

In the above subsection, we discussed the strong decay patterns of the S-wave hidden-charm tetraquarks under the heavy quark symmetry. We notice that some decays depend on the transition matrix elements between the initial and final states which either have different color configurations or belong to different color-spin  $SU(6)_{cs}$  representations. In this subsection, we discuss two kinds of suppressions.

Suppose the strong decays are induced by the general interaction which includes the Coulomb interaction, the linear confinement and the color-magnetic interaction,

$$V_{eff} = - \sum_{i>j} \frac{a_{ij}}{|r_i - r_j|} + \sum_{i>j} b_{ij}|r_i - r_j| - \sum_{i>j} c_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (27)$$

where  $a_{ij}$  and  $b_{ij}$  are coefficients depending on  $\vec{\lambda}_i \cdot \vec{\lambda}_j$ . Their specific values are not important for the following discussions. Generally, the coefficient  $c_{ij} \propto 1/(m_i m_j)$ . In the heavy quark limit,  $m_c \rightarrow \infty$ , the color-magnetic interaction between two light quarks is dominant. Since both  $\vec{\lambda}_i$  and  $\vec{\lambda}_i \vec{\sigma}_i$  are generators of the  $SU(6)_{cs}$  color-spin group, these operators will not change the  $SU(6)_{cs}$  representations. In other words, the strong decay Hamiltonian is invariant under the color-spin  $SU(6)_{cs}$  transformation, which requires that the color-spin  $SU(6)_{cs}$  representations between the two light quarks be conserved in the strong decays. Therefore, the transition between different  $SU(6)_{cs}$  representations is strongly suppressed in the heavy quark symmetry limit.

Similarly, the transition between different color configurations is also suppressed since such a process involves the exchange of the soft gluons. This suppression may not be so strong as the suppression due to the violation of the color-spin symmetry.

With the above analysis, we can simplify the decay matrix elements of the S-wave hidden charm tetraquarks further, which are shown in Table III. We use  $\epsilon$  to denote the suppressed decay matrix due to the non-conservation of the color-spin  $SU(6)_{cs}$  representations. We use  $\bar{\epsilon}$  to denote the dually suppressed decay matrix due to the non-conservation of the color configuration and color-spin  $SU(6)_{cs}$  representation.

From Table III, the P-wave decay mode  $h_c \pi$  for all the  $1^+(1^{+-})$  tetraquarks is suppressed. The final state  $h_c \pi \{^3P_1\}$  is governed by the spin configuration  $(0_H \otimes 1_l)^{+-}$ , and  $\pi$  belongs to the color-spin  $SU(6)_{cs}$  singlet. In contrast, the spin configuration  $(0_H \otimes 1_l)^{+-}$  in the initial states arises only from the color-spin configuration  $(1, 1, 0)_H^+ \otimes (35, 1, 1)_l^{--}$  or  $(35, 8, 0)_H^+ \otimes (35, 8, 1)_l^{--}$ , both of which contain the color-spin  $35_{cs}$  representation only. Thus, the color-spin  $SU(6)_{cs}$  representations between two light quarks are not the same for the decay mode  $h_c \pi \{^3P_1\}$ . Therefore, their  $h_c \pi$  decay width is suppressed in the heavy quark limit. We want to emphasize that the decay mode  $h_c \pi \{^3P_1\}$  of the  $1^+(1^{+-})$  molecular states is not suppressed under the heavy quark symmetry as shown in Refs. [67, 68].

As shown in Table III, the decay modes  $J/\psi \pi$ ,  $\psi(1^3D_1)\pi$ ,  $\eta_c \rho$ ,  $\eta_{c2} \rho$  and  $h_c \pi \{^3P_1\}$  of the  $1^+(1^{+-})$  tetraquarks with the color-spin representations  $280_{cs}$  are all suppressed. The states  $|280_{cs}, 1_c, 1\rangle_{(21, \bar{15})}$  and  $|280_{cs}, 1_c, 1\rangle_{(15, \bar{21})}$  may have narrower widths than the states with the color-spin representations  $35_{cs}$ , which provides an effective way to distinguish the color-spin representation of the  $1^+(1^{+-})$  tetraquarks.

The P-wave decay modes  $\chi_{cJ} \pi \{^3P_1\}$  ( $J = 0, 1, 2$ ) are not allowed for the four kinds of  $1^-(1^{++})$  tetraquarks due to the non-conservation of the color-spin  $SU(6)_{cs}$  representations.

The decay modes  $J/\psi \eta$ ,  $\psi(1^3D_1)\eta$ ,  $\eta_c \omega$ ,  $\eta_{c2} \omega$  and  $h_c \sigma$  of the  $0^-(1^{+-})$  tetraquarks with color-spin representations  $280_{cs}$  are all suppressed, which is similar to the decays of its isovector partners into  $J/\psi \pi$ ,  $\psi(1^3D_1)\pi$ ,  $\eta_c \rho$ ,  $\eta_{c2} \rho$  and  $h_c \pi \{^3P_1\}$ . The final state  $h_c \sigma$  is governed by the spin configuration  $(0_H \otimes 1_l)^{+-}$  with the color-spin  $SU(6)_{cs}$  representation  $1_{cs}$ , which does not appear in the color-spin wave functions of all the  $0^-(1^{+-})$  tetraquarks. Therefore, their decay into  $h_c \sigma$  is suppressed in the heavy quark limit. However, the  $h_c \sigma$  mode of the  $0^-(1^{+-})$  molecular states is not suppressed [67, 68].

Since the isoscalar  $1^{++}$  tetraquarks contain the color-spin configurations  $(35, 8, 1)_H^- \otimes (35, 8, 1)_l^{--}$  and  $(35, 1, 1)_H^- \otimes (35, 1, 1)_l^{--}$  only, their decays into  $\chi_{c1} \sigma$  and  $\chi_{cJ} \eta \{^3P_1\}$  ( $J = 0, 1, 2$ ) are suppressed, which are dominated by the color-spin configuration  $(35, 1, 1)_H^- \otimes (1, 1, 1)_l^{--}$ . In contrast, these decay modes of the molecular states with the same quantum numbers are not suppressed. This discrepancy may be used to distinguish the inner structure of the  $0^+(1^{++})$  hidden-charm states.

From the Table III, we notice that all the  $1^-(0^{++})$  tetraquarks are not allowed to decay into  $\chi_{c1} \pi$  via P-wave.  $\chi_{c1} \pi \{^3P_0\}$  is governed by the color-spin configuration  $(35, 1, 1)_H^- \otimes (1, 1, 1)_l^{--}$  while the color-spin representation of all the initial states with the light spin  $S_l = 1$  is  $35_{cs}$ . Therefore, these decays are suppressed in the heavy quark limit. We also notice that  $\eta_c \pi$  is an allowed decay mode for the  $1^-(0^{++})$  tetraquarks with the color-spin  $SU(6)_{cs}$  representation  $1_{cs}$ , while this decay mode is not allowed for the  $1^-(0^{++})$  tetraquarks with the color-spin  $SU(6)_{cs}$  representation  $405_{cs}$  or  $189_{cs}$ .

The  $\eta_c \pi$  decay mode of the states  $|1_{cs}, 1_c, 0\rangle_{(21, \bar{21})}$  and  $|1_{cs}, 1_c, 0\rangle_{(15, \bar{15})}$  arises from both the color singlet and color octet components. The contribution from the color octet is suppressed due to the non-conservation of the color-spin representations. The  $\eta_c \pi$  decay mode of the initial states  $|405_{cs}, 1_c, 0\rangle_{(21, \bar{21})}$  and  $|189_{cs}, 1_c, 0\rangle_{(15, \bar{15})}$  contain the contribution from the color octet only. Thus, their decay into  $\eta_c \pi$  is suppressed, which provides a way to distinguish  $|405_{cs}, 1_c, 0\rangle_{(21, \bar{21})}$  and  $|189_{cs}, 1_c, 0\rangle_{(15, \bar{15})}$ .

For the  $0^+(0^{++})$  tetraquarks, the decay mode  $\eta_c \eta$  is allowed for the states  $|1_{cs}, 1_c, 0\rangle_{(21, \bar{21})}$  and  $|1_{cs}, 1_c, 0\rangle_{(15, \bar{15})}$ . Both the color singlet and color octet contribute to the  $\eta_c \eta$  decay width. The decay mode  $\eta_c \eta$  is not allowed for the states  $|405_{cs}, 1_c, 0\rangle_{(21, \bar{21})}$  and  $|189_{cs}, 1_c, 0\rangle_{(15, \bar{15})}$ . Only the color octet contributes to this decay. This is similar to the decay patterns of their isovector partners.

All the four  $0^+(0^{++})$  states are not allowed to decay into  $\chi_{c0}\sigma$  and  $\chi_{c1}\eta$   $\{^3P_0\}$ . The color-spin configuration  $(35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{--}$  is dominant for these decay modes. This color-spin configuration does not appear in all the four initial states. Again, this feature is different from the decay patterns of their molecular counterparts [67, 68].

#### IV. DECAY PATTERNS OF THE P-WAVE TETRAQUARKS

##### A. Color-spin structures of the P-wave tetraquarks

The P-wave excitation can exist either between the diquark and anti-diquark, or inside the diquark or anti-diquark. Here we use "type I" to denote the case where the P-wave excitation is between the diquark and anti-diquark pair, and "type II" to denote the case where the P-wave excitation is either inside the diquark or anti-diquark. We categorize the two kinds of situations into two subsections.

###### 1. The type I P-wave tetraquarks

According to the discussion in Sec. II, if the P-wave excitation exists between the diquark and anti-diquark pair, the P-wave will contribute to the C-parity of the system under C transformation. We have listed the color-spin wave functions of the P-wave tetraquarks in Appendix B. The type I P-wave tetraquarks with  $J^{PC} = 1^{--}$  have three kinds of configurations, which are  $1^{--}(^1P_1)$ ,  $1^{--}(^3P_1)$  and  $1^{--}(^5P_1)$ . The notation  $(^1P_1)$  means the spin of all the four quarks is 0, and the total spin couples with the P-wave into the total angular momentum 1.

There are four  $1^{--}(^1P_1)$  tetraquark states. Since the isospin wave functions do not affect the results of the color-spin rearrangement, their isoscalar and isovector states have the same color-spin wave functions. All the four states contain the color  $SU(3)_c$  singlet and octet, which is different from the color-spin wave functions of their molecular counterparts. The color octet terms in the color-spin wave functions of the states  $|1_{cs}, 1_c, 0\rangle_{(21, \bar{2}1)}$  and  $|1_{cs}, 1_c, 0\rangle_{(15, \bar{1}5)}$  have both the heavy spin singlet and heavy spin triplet. The color octet configurations of the states  $|405_{cs}, 1_c, 0\rangle_{(21, \bar{2}1)}$  and  $|189_{cs}, 1_c, 0\rangle_{(15, \bar{1}5)}$  have the heavy spin singlet only.

From the color-spin wave functions of the  $1^{--}(^3P_1)$  states, we notice that the states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  have the same color-spin wave functions. All the four tetraquarks have spin configurations such as  $(1_H \otimes 0_l)_1^{--}$ ,  $(1_H \otimes 1_l)_1^{--}$  and  $(1_H \otimes 2_l)_1^{--}$ . Each of these spin configurations includes the color singlet and octet. Nevertheless, none of the states contains the spin configuration  $(0_H \otimes 1_l)_1^{--}$ . This feature is different from the color-spin wave functions of the  $1^{--}(^1P_1)$  tetraquarks.

There are only two  $1^{--}(^5P_1)$  tetraquark states. Their color-spin wave functions are similar to those of  $1^{--}(^3P_1)$  states, which contain the spin configurations like  $(1_H \otimes 0_l)_1^{--}$ ,  $(1_H \otimes 1_l)_1^{--}$  and  $(1_H \otimes 2_l)_1^{--}$ , but do not have the spin configuration  $(0_H \otimes 1_l)_1^{--}$ .

The P-wave  $1^{++}$  tetraquarks of type I have six kinds of states. As shown in Appendix B, the states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  have the same color-spin wave functions, which contain the color singlet configuration only. The states  $|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$  and  $|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$  also have the same color-spin wave functions which contain the color octet configuration only.

There are four tetraquark states with  $J^{PC} = 0^{--}$  of type I. All the four states contain the spin configuration  $(1_H \otimes 1_l)_0^{--}$  only. But none of them has the spin configuration  $(0_H \otimes 0_l)_0^{--}$ . Their spin configuration  $(1_H \otimes 1_l)_0^{--}$  arises from both the color singlet and color octet. The states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  have the same color-spin wave functions. They have similar strong decay patterns without considering their phase space difference.

There are six  $0^{++}$  P-wave tetraquarks of type I. The state  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  has the same re-coupling coefficients with the state  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$ . Their spin configurations are from the color singlet only. The state  $|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$  also has the same re-coupling coefficients with the state  $|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$ . Both of them have the color octet configuration only.

###### 2. The type II P-wave tetraquarks

We have listed the color-spin wave functions of the type II P-wave tetraquarks in Appendix B. The P-wave tetraquarks with  $J^{PC} = 1^{--}$  have only one configuration  $1^{--}(^3P_1)$ , which have six kinds of states. The states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  have the same re-coupling coefficients. The state  $|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$  and  $|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$  also have the same re-coupling coefficients. Moreover, the states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  have the color octet configuration only, while the states  $|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$  and  $|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$  contain the color singlet configuration only. The states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$  contain not only the color singlet but also the color octet. However, none of them has the spin configuration  $(1_H \otimes 0_l)_1^{--}$ , which differs from the  $1^{--}$  P-wave tetraquarks of type I.

The  $1^{++}$  tetraquarks have three kinds of configurations, which are the  $1^{++}(^1P_1)$ ,  $1^{++}(^3P_1)$  and  $1^{++}(^5P_1)$ . The  $1^{++}(^1P_1)$  states have four kinds of states. Their color-spin wave functions include both the color singlet and color octet. The spin configuration

$(0_H \otimes 1_I)^{-+}$  of the states  $|1_{cs}, 1_c, 0\rangle_{(21, \bar{2}1)}$  and  $|1_{cs}, 1_c, 0\rangle_{(15, \bar{1}5)}$  comes from not only the color singlet terms but also from the color octet terms. The spin configuration  $(0_H \otimes 1_I)^{-+}$  of the states  $|405_{cs}, 1_c, 0\rangle_{(21, \bar{2}1)}$  and  $|189_{cs}, 1_c, 0\rangle_{(15, \bar{1}5)}$  comes from the color octet terms only. The obvious difference between the  $1^{--}({}^3P_1)$  states and  $1^{--}({}^1P_1)$  states is that none of the  $1^{--}({}^3P_1)$  states contains the spin configuration  $(0_H \otimes 1_I)^{-+}$ . We also notice that the states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  have the same color-spin wave functions, and the states  $|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$  and  $|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$  also have the same color-spin wave functions.

There are only two  $1^{--}({}^5P_1)$  tetraquark states. Neither of them contains the spin configuration  $(0_H \otimes 1_I)^{-+}$ , which is similar to the  $1^{--}({}^3P_1)$  states.

The P-wave  $0^{--}$  tetraquarks of type I include six kinds of different states. The state  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  has the same color-spin structures with the state  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$ . Their spin configurations arise from the color singlet only. The state  $|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$  also has the same color-spin structures with the state  $|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$ . Their spin configurations come from the color octet configuration only.

There are four tetraquark states with  $J^{PC} = 0^{-+}$  of type I. All of them contain the spin configuration  $(1_H \otimes 1_I)^{-+}$ . Nevertheless, none of them has the spin configuration  $(0_H \otimes 0_I)^{-+}$ . Their spin configuration  $(1_H \otimes 1_I)^{-+}$  comes not only from the color singlet but also from the color octet. The states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  also have the same color-spin wave functions.

## B. The strong decay matrix for P-wave tetraquarks

With the preparations in Sec. II, we are ready to discuss the strong decay behavior of the P-wave tetraquarks. The strong decay matrix elements of the P-wave tetraquarks are listed in Table IV, where the parameters  $A_{(1-24)}$  and  $B_{(1-24)}$  are defined in Appendix C. The P-wave tetraquarks of type I with  $J^{PC} = 1^{--}$  have three kinds of configurations, which are  $1^{--}({}^1P_1)$ ,  $1^{--}({}^3P_1)$  and  $1^{--}({}^5P_1)$ .

All the four  $1^{--}({}^1P_1)$  states of type I have the S-wave decay modes  $h_c\pi, \chi_{cJ}P$  ( $J = 0, 1, 2$ ),  $J/\psi a_0(980)$  and the P-wave decay modes  $J/\psi\pi \{{}^3P_1\}$ ,  $\psi(1^3D_1)\pi \{{}^3P_1\}$ ,  $\psi(1^3D_2)\pi \{{}^5P_1\}$ . The decay mode  $h_c\pi$  is dominated by the spin configuration  $(0_H \otimes 1_I)^{-+}$ . The spin configuration  $(0_H \otimes 1_I)^{-+}$  of the initial states  $|405_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|189_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  arises from the contributions of the color octet only. The initial states  $|1_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|1_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  decay into  $h_c\pi$  through both the color singlet and color octet. All the four  $1^{--}({}^1P_1)$  states can decay into  $J/\psi\pi$  via P-wave, which is governed by the spin configuration  $(1_H \otimes 1_I)^{-+}$ . The decay mode  $J/\psi a_0(980)$  is dominated by the spin configuration  $(1_H \otimes 0_I)^{-+}$ . Their decays into  $J/\psi a_0(980)$  have the contributions from both the color singlet and color octet. The decay modes  $\chi_{cJ}P$  ( $J = 0, 1, 2$ ) depend on the spin configurations  $(1_H \otimes 0_I)^{-+}$ ,  $(1_H \otimes 1_I)^{-+}$  and  $(1_H \otimes 2_I)^{-+}$ . The P-wave decay modes  $\psi(1^3D_1)\pi \{{}^3P_1\}$  and  $\psi(1^3D_2)\pi \{{}^5P_1\}$  both depend on the spin configurations  $(1_H \otimes 1_I)^{-+}$  and  $(1_H \otimes 2_I)^{-+}$ . There is obvious difference between tetraquarks with configurations  $1^{--}({}^1P_1)$  and  $1^{--}({}^3P_1)$  of type I. All the  $1^{--}({}^1P_1)$  states can decay into  $h_c\pi$  while all the  $1^{--}({}^3P_1)$  of type I can not. The  $1^{--}({}^1P_1)$  and  $1^{--}({}^3P_1)$  tetraquarks of type I have the common decay modes  $\chi_{cJ}P$  ( $J = 0, 1, 2$ ),  $J/\psi a_0(980)$ ,  $J/\psi\pi \{{}^3P_1\}$ ,  $\psi(1^3D_1)\pi \{{}^3P_1\}$  and  $\psi(1^3D_2)\pi \{{}^5P_1\}$ .

The  $1^{--}({}^5P_1)$  states of type I can not decay into  $h_c\pi$  in the heavy quark limit, which provides a way to distinguish the  $1^{--}({}^1P_1)$  states from  $1^{--}({}^3P_1)$  and  $1^{--}({}^5P_1)$ . The  $1^{--}({}^5P_1)$  states also have the S-wave decay modes  $\chi_{cJ}P$  ( $J = 0, 1, 2$ ),  $J/\psi a_0(980)$ , and the P-wave decay modes  $J/\psi\pi \{{}^3P_1\}$ ,  $\psi(1^3D_1)\pi \{{}^3P_1\}$  and  $\psi(1^3D_2)\pi \{{}^5P_1\}$ . The  $1^{--}({}^3P_1)$  states of type II have the S-wave decay mode  $h_c\pi$  and the P-wave decay mode  $J/\psi\pi$ , where the decays from the states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  are dominated by the color singlet, while the decays from the states  $|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$  and  $|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$  are dominated by the color octet. The above decays from the states  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$  are governed not only by the color singlet but also the color octet. They can also decay into  $\chi_{cJ}P$  ( $J = 0, 1, 2$ ),  $\psi(1^3D_1)\pi \{{}^3P_1\}$  and  $\psi(1^3D_2)\pi \{{}^5P_1\}$ , where the spin configurations  $(1_H \otimes 1_I)^{-+}$  and  $(1_H \otimes 2_I)^{-+}$  are dominant. Their decay mode  $J/\psi a_0(980)$  is suppressed. The final state  $J/\psi a_0(980)$  contains the spin configuration  $(1_H \otimes 0_I)^{-+}$  only, which does not appear in all the initial  $1^{--}({}^3P_1)$  states of type II. Therefore, their decays into  $J/\psi a_0(980)$  are strongly suppressed under the heavy quark symmetry.

If the P-wave excitation is between the diquark and anti-diquark pair, the  $0^{--}({}^1P_1)$  tetraquarks and their isovector partners have three kinds of configurations  $1^{--}({}^1P_1)$ ,  $1^{--}({}^3P_1)$  and  $1^{--}({}^5P_1)$ .  $J/\psi\sigma$ ,  $\psi(1^3D_1)\sigma$ ,  $h_c\eta$ ,  $\chi_{cJ}\omega$  ( $J = 0, 1, 2$ ),  $J/\psi\eta \{{}^3P_1\}$ ,  $\psi(1^3D_1)\eta \{{}^3P_1\}$  and  $\psi(1^3D_2)\eta \{{}^5P_1\}$  are the allowed decay modes of the  $1^{--}({}^1P_1)$  tetraquarks of type I. The  $J/\psi\sigma$  is governed by the spin configuration  $(1_H \otimes 0_I)^{-+}$ . The decay mode  $\psi(1^3D_1)\sigma$  is governed by the spin configuration  $(1_H \otimes 2_I)^{-+}$ . The P-wave decay mode  $J/\psi\eta$  is dominated by the spin configuration  $(1_H \otimes 1_I)^{-+}$ . The spin configuration  $(0_H \otimes 1_I)^{-+}$  is dominant for the decay mode  $h_c\eta$ . The spin configuration  $(0_H \otimes 1_I)^{-+}$  of the initial states  $|405_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|189_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  arises from the color octet only. The initial states  $|1_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|1_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  decay into  $h_c\pi$  via both the color singlet and color octet. The decay modes  $\chi_{cJ}\omega$  ( $J = 0, 1, 2$ ) depend on the spin configurations  $(1_H \otimes 0_I)^{-+}$ ,  $(1_H \otimes 1_I)^{-+}$  and  $(1_H \otimes 2_I)^{-+}$ . The isoscalar  $1^{--}({}^3P_1)$  and  $1^{--}({}^5P_1)$  tetraquarks of type I also have the decay modes  $J/\psi\sigma$ ,  $\psi(1^3D_1)\sigma$ ,  $\chi_{cJ}\omega$  ( $J = 0, 1, 2$ ),  $J/\psi\eta \{{}^3P_1\}$ ,  $\psi(1^3D_1)\eta \{{}^3P_1\}$  and  $\psi(1^3D_2)\eta \{{}^5P_1\}$ , where the spin configurations which contribute to their decays are similar to those of the  $1^{--}({}^1P_1)$  tetraquarks of type I. We notice that the decay mode  $h_c\eta$  is suppressed for the  $1^{--}({}^3P_1)$  and  $1^{--}({}^5P_1)$  tetraquarks, while it is allowed for the  $1^{--}({}^1P_1)$  tetraquarks of type I. This feature provides a way to distinguish the  $1^{--}({}^1P_1)$  tetraquarks from the  $1^{--}({}^3P_1)$  and  $1^{--}({}^5P_1)$  tetraquarks.

The  $1^{--}({}^3P_1)$  tetraquarks of type II can also decay into  $\chi_{cJ}\omega$  ( $J = 0, 1, 2$ ),  $\psi(1^3D_1)\eta \{{}^3P_1\}$  and  $\psi(1^3D_2)\eta \{{}^5P_1\}$ , where the spin



TABLE IV: The decay matrix elements of the tetraquarks with  $J^{PC} = 1^{--}$ . The reduced matrix elements  $H_\alpha^{ij} \propto \langle Q, i || H_{eff}(\alpha) || j \rangle$ , where the indices  $i$  and  $j$  denote the light spin of the final and initial hadron respectively, and  $Q$  is the angular momentum of the final light meson. The quantum numbers in the bracelets represent the total angular momentum configurations of the final state particles.

$1^+(1^{--})[{}^1P_1]$			Final state					
Type I	$h_c\pi$	$\chi_{c0}\rho$	$\chi_{c1}\rho$	$\chi_{c2}\rho$	$J/\psi\pi\{^3P_1\}$	$J/\psi a_0(980)$	$\psi(1^3D_1)\pi/\psi(1^3D_2)\pi$	
$ 1_{cs}, 1_c, 1\rangle_{(21,21)}$	$\frac{\sqrt{21}}{6}H_\pi^{11} + \frac{\sqrt{42}}{21}\tilde{H}_\pi^{11}$	$A_1$	$A_2$	$A_3$	$\frac{\sqrt{21}}{42}H_\pi^{01} + \frac{\sqrt{21}}{42}\tilde{H}_\pi^{01}$	$-\frac{\sqrt{7}}{42}H_{a0}^{00} - \frac{\sqrt{7}}{42}\tilde{H}_{a0}^{00}$	$B_1/B_2$	
$ 405_{cs}, 1_c, 1\rangle_{(21,21)}$	$\frac{3\sqrt{7}}{14}\tilde{H}_\pi^{11}$	$A_4$	$A_5$	$A_6$	$\frac{2\sqrt{14}}{21}H_\pi^{01} - \frac{5\sqrt{7}}{42}\tilde{H}_\pi^{01}$	$-\frac{2\sqrt{42}}{63}H_{a0}^{00} - \frac{5\sqrt{21}}{126}\tilde{H}_{a0}^{00}$	$B_3/B_4$	
$ 1_{cs}, 1_c, 1\rangle_{(15,15)}$	$\frac{\sqrt{15}}{6}H_\pi^{11} - \frac{\sqrt{30}}{15}\tilde{H}_\pi^{11}$	$A_7$	$A_8$	$A_9$	$-\frac{\sqrt{15}}{30}H_\pi^{01} - \frac{\sqrt{30}}{15}\tilde{H}_\pi^{01}$	$-\frac{\sqrt{5}}{30}H_{a0}^{00} + \frac{\sqrt{10}}{15}\tilde{H}_{a0}^{00}$	$B_5/B_6$	
$ 189_{cs}, 1_c, 1\rangle_{(15,15)}$	$-\frac{3\sqrt{5}}{10}\tilde{H}_\pi^{11}$	$A_{10}$	$A_{11}$	$A_{12}$	$\frac{2\sqrt{10}}{15}H_\pi^{01} + \frac{\sqrt{5}}{30}\tilde{H}_\pi^{01}$	$-\frac{2\sqrt{30}}{45}H_{a0}^{00} - \frac{\sqrt{15}}{90}\tilde{H}_{a0}^{00}$	$B_7/B_8$	
$1^+(1^{--})[{}^3P_1]$			Final state					
Type I	$h_c\pi$	$\chi_{c0}\rho$	$\chi_{c1}\rho$	$\chi_{c2}\rho$	$J/\psi\pi\{^3P_1\}$	$J/\psi a_0(980)$	$\psi(1^3D_1)\pi/\psi(1^3D_2)\pi$	
$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	0	$A_{13}$	$A_{14}$	$A_{15}$	$-\frac{1}{6}H_\pi^{01} - \frac{\sqrt{2}}{3}\tilde{H}_\pi^{01}$	$\frac{\sqrt{3}}{9}H_{a0}^{00} + \frac{2\sqrt{6}}{9}\tilde{H}_{a0}^{00}$	$B_9/B_{10}$	
$ 35_{cs}, 1_c, 1\rangle_{(15,15)}$	0	$A_{13}$	$A_{14}$	$A_{15}$	$-\frac{1}{6}H_\pi^{01} - \frac{\sqrt{2}}{3}\tilde{H}_\pi^{01}$	$\frac{\sqrt{3}}{9}H_{a0}^{00} + \frac{2\sqrt{6}}{9}\tilde{H}_{a0}^{00}$	$B_9/B_{10}$	
$ 280_{cs}, 1_c, 1\rangle_{(21,15)}$	0	$A_{16}$	$A_{17}$	$A_{18}$	$\frac{\sqrt{2}}{3}H_\pi^{01} - \frac{1}{6}\tilde{H}_\pi^{01}$	$-\frac{2\sqrt{6}}{9}H_{a0}^{00} + \frac{\sqrt{3}}{9}\tilde{H}_{a0}^{00}$	$B_{11}/B_{12}$	
$ 280_{cs}, 1_c, 1\rangle_{(15,21)}$	0	$A_{16}$	$A_{17}$	$A_{18}$	$\frac{\sqrt{2}}{3}H_\pi^{01} - \frac{1}{6}\tilde{H}_\pi^{01}$	$-\frac{2\sqrt{6}}{9}H_{a0}^{00} + \frac{\sqrt{3}}{9}\tilde{H}_{a0}^{00}$	$B_{11}/B_{12}$	
$1^+(1^{--})[{}^5P_1]$			Final state					
Type I	$h_c\pi$	$\chi_{c0}\rho$	$\chi_{c1}\rho$	$\chi_{c2}\rho$	$J/\psi\pi\{^3P_1\}$	$J/\psi a_0(980)$	$\psi(1^3D_1)\pi/\psi(1^3D_2)\pi$	
$ 405_{cs}, 1_c, 1\rangle_{(21,21)}$	0	$A_{19}$	$A_{20}$	$A_{21}$	$\frac{\sqrt{10}}{6}H_\pi^{01} + \frac{\sqrt{5}}{6}\tilde{H}_\pi^{01}$	$\frac{\sqrt{30}}{9}H_{a0}^{00} + \frac{\sqrt{15}}{9}\tilde{H}_{a0}^{00}$	$B_{13}/B_{14}$	
$ 189_{cs}, 1_c, 1\rangle_{(15,15)}$	0	$A_{22}$	$A_{23}$	$A_{24}$	$-\frac{\sqrt{5}}{6}H_\pi^{01} + \frac{\sqrt{10}}{6}\tilde{H}_\pi^{01}$	$-\frac{\sqrt{15}}{9}H_{a0}^{00} + \frac{\sqrt{30}}{9}\tilde{H}_{a0}^{00}$	$B_{15}/B_{16}$	
$1^+(1^{--})[{}^3P_1]$			Final state					
Type II	$h_c\pi$	$\chi_{c0}\rho$	$\chi_{c1}\rho$	$\chi_{c2}\rho$	$J/\psi\pi\{^3P_1\}$	$J/\psi a_0(980)$	$\psi(1^3D_1)\pi/\psi(1^3D_2)\pi$	
$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	$-\frac{\sqrt{5}}{2}H_\pi^{11}$	$-\frac{\sqrt{6}}{6}H_\rho^{11}$	$\frac{\sqrt{2}}{4}H_\rho^{11}$	$\frac{\sqrt{30}}{12}H_\rho^{11}$	$\frac{\sqrt{5}}{2}H_\pi^{01}$	0	$B_{17}/B_{18}$	
$ 35_{cs}, 1_c, 1\rangle_{(15,15)}$	$-\frac{\sqrt{2}}{2}H_\pi^{11}$	$-\frac{\sqrt{6}}{6}H_\rho^{11}$	$\frac{\sqrt{2}}{4}H_\rho^{11}$	$\frac{\sqrt{10}}{12}H_\rho^{11}$	$\frac{\sqrt{2}}{2}H_\pi^{01}$	0	$B_{17}/B_{18}$	
$ 35_{cs}, 1_c, 1\rangle_{(21,15)}$	$\frac{\sqrt{3}}{6}H_\pi^{11} + \frac{\sqrt{6}}{6}\tilde{H}_\pi^{11}$	$-\frac{1}{3}H_\rho^{11} - \frac{\sqrt{2}}{6}\tilde{H}_\rho^{11}$	$\frac{\sqrt{3}}{6}H_\rho^{11} + \frac{\sqrt{6}}{12}\tilde{H}_\rho^{11}$	$\frac{\sqrt{5}}{6}H_\rho^{11} + \frac{\sqrt{10}}{12}\tilde{H}_\rho^{11}$	$\frac{\sqrt{3}}{6}H_\pi^{01} + \frac{\sqrt{6}}{6}\tilde{H}_\pi^{01}$	0	$B_{19}/B_{20}$	
$ 35_{cs}, 1_c, 1\rangle_{(15,21)}$	$\frac{\sqrt{6}}{6}H_\pi^{11} - \frac{\sqrt{3}}{3}\tilde{H}_\pi^{11}$	$-\frac{\sqrt{2}}{6}H_\rho^{11} + \frac{1}{3}\tilde{H}_\rho^{11}$	$\frac{\sqrt{6}}{12}H_\rho^{11} - \frac{\sqrt{3}}{6}\tilde{H}_\rho^{11}$	$\frac{\sqrt{10}}{12}H_\rho^{11} - \frac{\sqrt{5}}{6}\tilde{H}_\rho^{11}$	$\frac{\sqrt{6}}{6}H_\pi^{01} - \frac{\sqrt{3}}{3}\tilde{H}_\pi^{01}$	0	$B_{21}/B_{22}$	
$ 280_{cs}, 1_c, 1\rangle_{(21,15)}$	$\frac{\sqrt{2}}{2}\tilde{H}_\pi^{11}$	$-\frac{\sqrt{6}}{6}\tilde{H}_\rho^{11}$	$\frac{\sqrt{2}}{4}\tilde{H}_\rho^{11}$	$\frac{\sqrt{30}}{12}\tilde{H}_\rho^{11}$	$-\frac{\sqrt{2}}{2}\tilde{H}_\pi^{01}$	0	$B_{23}/B_{24}$	
$ 280_{cs}, 1_c, 1\rangle_{(15,21)}$	$\frac{\sqrt{2}}{2}\tilde{H}_\pi^{11}$	$-\frac{\sqrt{6}}{6}\tilde{H}_\rho^{11}$	$\frac{\sqrt{2}}{4}\tilde{H}_\rho^{11}$	$\frac{\sqrt{30}}{12}\tilde{H}_\rho^{11}$	$-\frac{\sqrt{2}}{2}\tilde{H}_\pi^{01}$	0	$B_{23}/B_{24}$	
$0^-(1^{--})[{}^1P_1]$			Final state					
Type I	$J/\psi\sigma$	$\psi(1^3D_1)\sigma$	$h_c\eta$	$\chi_{c0}\omega$	$\chi_{c1}\omega$	$\chi_{c2}\omega$	$\psi(1^3D_1)\eta/\psi(1^3D_2)\eta$	$J/\psi\eta\{^3P_1\}$
$ 1_{cs}, 1_c, 1\rangle_{(21,21)}$	$-\frac{\sqrt{7}}{42}H_\sigma^{01} - \frac{\sqrt{7}}{42}\tilde{H}_\sigma^{01}$	$-\frac{\sqrt{35}}{42}H_\sigma^{21} - \frac{\sqrt{35}}{42}\tilde{H}_\sigma^{21}$	$\frac{\sqrt{21}}{6}H_\eta^{11} + \frac{\sqrt{42}}{21}\tilde{H}_\eta^{11}$	$A_1$	$A_2$	$A_3$	$B_1/B_2$	$\frac{\sqrt{21}}{42}H_\eta^{01} + \frac{\sqrt{21}}{42}\tilde{H}_\eta^{01}$
$ 405_{cs}, 1_c, 1\rangle_{(21,21)}$	$\frac{2\sqrt{42}}{63}H_\sigma^{01} + \frac{5\sqrt{21}}{126}\tilde{H}_\sigma^{01}$	$-\frac{2\sqrt{10}}{63}H_\sigma^{21} + \frac{5\sqrt{105}}{126}\tilde{H}_\sigma^{21}$	$\frac{3\sqrt{7}}{14}\tilde{H}_\eta^{11}$	$A_4$	$A_5$	$A_6$	$B_3/B_4$	$\frac{2\sqrt{14}}{21}H_\eta^{01} - \frac{5\sqrt{7}}{42}\tilde{H}_\eta^{01}$
$ 1_{cs}, 1_c, 1\rangle_{(15,15)}$	$\frac{\sqrt{5}}{30}H_\sigma^{01} + \frac{\sqrt{10}}{15}\tilde{H}_\sigma^{01}$	$\frac{1}{6}H_\sigma^{21} + \frac{\sqrt{2}}{3}\tilde{H}_\sigma^{21}$	$\frac{\sqrt{15}}{6}H_\eta^{11} - \frac{\sqrt{30}}{15}\tilde{H}_\eta^{11}$	$A_7$	$A_8$	$A_9$	$B_5/B_6$	$-\frac{\sqrt{15}}{30}H_\eta^{01} - \frac{\sqrt{30}}{15}\tilde{H}_\eta^{01}$
$ 189_{cs}, 1_c, 1\rangle_{(15,15)}$	$\frac{2\sqrt{30}}{45}H_\sigma^{01} - \frac{\sqrt{15}}{90}\tilde{H}_\sigma^{01}$	$-\frac{2\sqrt{6}}{9}H_\sigma^{21} - \frac{\sqrt{3}}{18}\tilde{H}_\sigma^{21}$	$-\frac{3\sqrt{5}}{10}\tilde{H}_\eta^{11}$	$A_{10}$	$A_{11}$	$A_{12}$	$B_7/B_8$	$\frac{2\sqrt{10}}{15}H_\eta^{01} + \frac{\sqrt{5}}{30}\tilde{H}_\eta^{01}$
$0^-(1^{--})[{}^3P_1]$			Final state					
Type I	$J/\psi\sigma$	$\psi(1^3D_1)\sigma$	$h_c\eta$	$\chi_{c0}\omega$	$\chi_{c1}\omega$	$\chi_{c2}\omega$	$\psi(1^3D_1)\eta/\psi(1^3D_2)\eta$	$J/\psi\eta\{^3P_1\}$
$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	$\frac{\sqrt{3}}{9}H_\sigma^{01} + \frac{2\sqrt{6}}{9}\tilde{H}_\sigma^{01}$	$-\frac{\sqrt{15}}{18}H_\sigma^{21} - \frac{\sqrt{30}}{9}\tilde{H}_\sigma^{21}$	0	$A_{13}$	$A_{14}$	$A_{15}$	$B_9/B_{10}$	$-\frac{1}{6}H_\eta^{01} - \frac{\sqrt{2}}{3}\tilde{H}_\eta^{01}$
$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	$\frac{\sqrt{3}}{9}H_\sigma^{01} + \frac{2\sqrt{6}}{9}\tilde{H}_\sigma^{01}$	$-\frac{\sqrt{15}}{18}H_\sigma^{21} - \frac{\sqrt{30}}{9}\tilde{H}_\sigma^{21}$	0	$A_{13}$	$A_{14}$	$A_{15}$	$B_9/B_{10}$	$-\frac{1}{6}H_\eta^{01} - \frac{\sqrt{2}}{3}\tilde{H}_\eta^{01}$
$ 280_{cs}, 1_c, 1\rangle_{(21,15)}$	$-\frac{2\sqrt{6}}{9}H_\sigma^{01} + \frac{\sqrt{3}}{9}\tilde{H}_\sigma^{01}$	$\frac{\sqrt{30}}{9}H_\sigma^{21} - \frac{\sqrt{15}}{18}\tilde{H}_\sigma^{21}$	0	$A_{16}$	$A_{17}$	$A_{18}$	$B_{11}/B_{12}$	$\frac{\sqrt{2}}{3}H_\eta^{01} - \frac{1}{6}\tilde{H}_\eta^{01}$
$ 280_{cs}, 1_c, 1\rangle_{(15,21)}$	$-\frac{2\sqrt{6}}{9}H_\sigma^{01} + \frac{\sqrt{3}}{9}\tilde{H}_\sigma^{01}$	$\frac{\sqrt{30}}{9}H_\sigma^{21} - \frac{\sqrt{15}}{18}\tilde{H}_\sigma^{21}$	0	$A_{16}$	$A_{17}$	$A_{18}$	$B_{11}/B_{12}$	$\frac{\sqrt{2}}{3}H_\eta^{01} - \frac{1}{6}\tilde{H}_\eta^{01}$
$0^-(1^{--})[{}^5P_1]$			Final state					
Type I	$J/\psi\sigma$	$\psi(1^3D_1)\sigma$	$h_c\eta$	$\chi_{c0}\omega$	$\chi_{c1}\omega$	$\chi_{c2}\omega$	$\psi(1^3D_1)\eta/\psi(1^3D_2)\eta$	$J/\psi\eta\{^3P_1\}$
$ 405_{cs}, 1_c, 1\rangle_{(21,21)}$	$\frac{\sqrt{30}}{9}H_\sigma^{01} + \frac{\sqrt{15}}{9}\tilde{H}_\sigma^{01}$	$\frac{\sqrt{6}}{18}H_\sigma^{21} + \frac{\sqrt{3}}{18}\tilde{H}_\sigma^{21}$	0	$A_{19}$	$A_{20}$	$A_{21}$	$B_{13}/B_{14}$	$\frac{\sqrt{10}}{6}H_\eta^{01} + \frac{\sqrt{5}}{6}\tilde{H}_\eta^{01}$
$ 189_{cs}, 1_c, 1\rangle_{(15,15)}$	$-\frac{\sqrt{15}}{9}H_\sigma^{01} + \frac{\sqrt{30}}{9}\tilde{H}_\sigma^{01}$	$-\frac{\sqrt{3}}{18}H_\sigma^{21} + \frac{\sqrt{6}}{18}\tilde{H}_\sigma^{21}$	0	$A_{22}$	$A_{23}$	$A_{24}$	$B_{15}/B_{16}$	$-\frac{\sqrt{5}}{6}H_\eta^{01} + \frac{\sqrt{10}}{6}\tilde{H}_\eta^{01}$
$0^-(1^{--})[{}^3P_1]$			Final state					
Type II	$J/\psi\sigma$	$\psi(1^3D_1)\sigma$	$h_c\eta$	$\chi_{c0}\omega$	$\chi_{c1}\omega$	$\chi_{c2}\omega$	$\psi(1^3D_1)\eta/\psi(1^3D_2)\eta$	$J/\psi\eta\{^3P_1\}$
$ 35_{cs}, 1_c, 1\rangle_{(21,21)}$	0	0	$-\frac{\sqrt{5}}{2}H_\eta^{11}$	$-\frac{\sqrt{6}}{6}H_\omega^{11}$	$\frac{\sqrt{5}}{4}H_\omega^{11}$	$\frac{\sqrt{30}}{12}H_\omega^{11}$	$B_{17}/B_{18}$	$\frac{\sqrt{5}}{2}H_\eta^{01}$
$ 35_{cs}, 1_c, 1\rangle_{(15,15)}$	0	0	$-\frac{\sqrt{2}}{2}H_\eta^{11}$	$-\frac{\sqrt{6}}{6}H_\omega^{11}$	$\frac{\sqrt{2}}{4}H_\omega^{11}$	$\frac{\sqrt{10}}{12}H_\omega^{11}$	$B_{17}/B_{18}$	$\frac{\sqrt{2}}{2}H_\eta^{01}$
$ 35_{cs}, 1_c, 1\rangle_{(21,15)}$	0	0	$\frac{\sqrt{3}}{6}H_\eta^{11} + \frac{\sqrt{6}}{6}\tilde{H}_\eta^{11}$	$-\frac{1}{3}H_\omega^{11} - \frac{\sqrt{2}}{6}\tilde{H}_\omega^{11}$	$\frac{\sqrt{3}}{6}H_\omega^{11} + \frac{\sqrt{6}}{12}\tilde{H}_\omega^{11}$	$\frac{\sqrt{5}}{6}H_\omega^{11} + \frac{\sqrt{10}}{12}\tilde{H}_\omega^{11}$	$B_{19}/B_{20}$	$\frac{\sqrt{3}}{3}H_\eta^{01} + \frac{\sqrt{6}}{6}\tilde{H}_\eta^{01}$
$ 35_{cs}, 1_c, 1\rangle_{(15,21)}$	0	0	$\frac{\sqrt{6}}{6}H_\eta^{11} - \frac{\sqrt{3}}{3}\tilde{H}_\eta^{11}$	$-\frac{\sqrt{2}}{6}H_\omega^{11} + \frac{1}{3}\tilde{H}_\omega^{11}$	$\frac{\sqrt{6}}{12}H_\omega^{11} - \frac{\sqrt{3}}{6}\tilde{H}_\omega^{11}$	$\frac{\sqrt{10}}{12}H_\omega^{11} - \frac{\sqrt{5}}{6}\tilde{H}_\omega^{11}$	$B_{21}/B_{22}$	$\frac{\sqrt{6}}{6}H_\eta^{01} - \frac{\sqrt{3}}{3}\tilde{H}_\eta^{01}$
$ 280_{cs}, 1_c, 1\rangle_{(21,15)}$	0	0	$\frac{\sqrt{2}}{2}\tilde{H}_\eta^{11}$	$-\frac{\sqrt{6}}{6}\tilde{H}_\omega^{11}$	$\frac{\sqrt{2}}{4}\tilde{H}_\omega^{11}$	$\frac{\sqrt{10}}{12}\tilde{H}_\omega^{11}$	$B_{23}/B_{24}$	$-\frac{\sqrt{2}}{2}\tilde{H}_\eta^{01}$
$ 280_{cs}, 1_c, 1\rangle_{(15,21)}$	0	0	$\frac{\sqrt{2}}{2}\tilde{H}_\eta^{11}$	$-\frac{\sqrt{6}}{6}\tilde{H}_\omega^{11}$	$\frac{\sqrt{2}}{4}\tilde{H}_\omega^{11}$	$\frac{\sqrt{10}}{12}\tilde{H}_\omega^{11}$	$B_{23}/B_{24}$	$-\frac{\sqrt{2}}{2}\tilde{H}_\eta^{01}$

configurations  $(1_H \otimes 1_\eta)_0^{--}$  and  $(1_H \otimes 2_\eta)_0^{--}$  are dominant. Similar to the case of the  $1^{--}({}^1P_1)$  tetraquarks of type I, the  $1^{--}({}^3P_1)$  tetraquarks of type II also have the S-wave decay mode  $h_c\eta$  and the P-wave decay mode  $J/\psi\eta [{}^3P_1]$ , which are governed by the spin configurations  $(0_H \otimes 1_\eta)_0^{--}$  and  $(1_H \otimes 1_\eta)_0^{--}$  respectively. The states  $|35_{cs}, 1_c, 1\rangle_{(21,21)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15,15)}$  contain the color singlet only while the states  $|280_{cs}, 1_c, 1\rangle_{(21,15)}$  and  $|280_{cs}, 1_c, 1\rangle_{(15,21)}$  contain the color octet only. Their decays into  $h_c\eta$  and  $J/\psi\eta [{}^3P_1]$  depend on the color singlet and the color octet respectively.

In Table IV, we notice that the  $1^{--}$  tetraquarks of type I differ from the  $1^{--}$  tetraquarks of type II greatly in the decay modes



$J/\psi\sigma$  and  $\psi(1^3D_1)\sigma$ . The isoscalar  $1^{--}$  tetraquarks of type I can decay into  $J/\psi\sigma$  and  $\psi(1^3D_1)\sigma$ . However, the isoscalar  $1^{--}$  tetraquarks of type II do not decay into these modes under the heavy quark symmetry.

TABLE V: The decay matrix elements of the tetraquarks with  $J^{PC} = 1^{--}$ . The reduced matrix elements  $H_{\alpha}^{ij} \propto \langle Q, i || H_{eff}(\alpha) || j \rangle$ , where the indices  $i$  and  $j$  denote the light spin of the final and initial hadron respectively, and  $Q$  is the angular momentum of the final light meson. The quantum numbers in the bracelets represent the total angular momentum configurations of the final state particles.

$I^G(J^{PC})$	Initial state	Final state			
$1^-(1^{--})$	Type I	$\chi_{c1}\pi$	$h_c\rho$	$\eta_c\pi \{^1P_1\}$	$\eta_{c2}\pi \{^5P_1\}$
	$ 35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$	$\frac{\sqrt{2}}{2} H_{\pi}^{11}$	$-\frac{\sqrt{2}}{2} H_{\rho}^{11}$	$-\frac{\sqrt{2}}{2} H_{\pi}^{01}$	$-\frac{\sqrt{2}}{2} H_{\pi}^{21}$
	$ 35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$	$\frac{\sqrt{2}}{2} H_{\pi}^{11}$	$-\frac{\sqrt{2}}{2} H_{\rho}^{11}$	$-\frac{\sqrt{2}}{2} H_{\pi}^{01}$	$-\frac{\sqrt{2}}{2} H_{\pi}^{21}$
	$ 35_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$	$\frac{\sqrt{3}}{3} H_{\pi}^{11} + \frac{\sqrt{6}}{6} \tilde{H}_{\pi}^{11}$	$\frac{\sqrt{3}}{3} H_{\rho}^{11} + \frac{\sqrt{6}}{6} \tilde{H}_{\rho}^{11}$	$\frac{\sqrt{3}}{3} H_{\pi}^{01} + \frac{\sqrt{6}}{6} \tilde{H}_{\pi}^{01}$	$\frac{\sqrt{3}}{3} H_{\pi}^{21} + \frac{\sqrt{6}}{6} \tilde{H}_{\pi}^{21}$
	$ 35_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$	$\frac{\sqrt{6}}{6} H_{\pi}^{11} - \frac{\sqrt{3}}{3} \tilde{H}_{\pi}^{11}$	$\frac{\sqrt{6}}{6} H_{\rho}^{11} - \frac{\sqrt{3}}{3} \tilde{H}_{\rho}^{11}$	$\frac{\sqrt{6}}{6} H_{\pi}^{01} - \frac{\sqrt{3}}{3} \tilde{H}_{\pi}^{01}$	$\frac{\sqrt{6}}{6} H_{\pi}^{21} - \frac{\sqrt{3}}{3} \tilde{H}_{\pi}^{21}$
	$ 280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$	$-\frac{\sqrt{2}}{2} \tilde{H}_{\pi}^{11}$	$\frac{\sqrt{2}}{2} \tilde{H}_{\rho}^{11}$	$\frac{\sqrt{2}}{2} \tilde{H}_{\pi}^{01}$	$\frac{\sqrt{2}}{2} \tilde{H}_{\pi}^{21}$
$1^-(1^{--})$	Type II	$\chi_{c1}\pi$	$h_c\rho$	$\eta_c\pi \{^1P_1\}$	$\eta_{c2}\pi \{^5P_1\}$
	$ 1_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$	$\frac{\sqrt{21}}{42} H_{\pi}^{11} + \frac{\sqrt{21}}{42} \tilde{H}_{\pi}^{11}$	$\frac{\sqrt{21}}{6} H_{\rho}^{11} + \frac{\sqrt{21}}{21} \tilde{H}_{\rho}^{11}$	$\frac{\sqrt{21}}{6} H_{\pi}^{01} + \frac{\sqrt{21}}{21} \tilde{H}_{\pi}^{01}$	$\frac{\sqrt{21}}{6} H_{\pi}^{21} + \frac{\sqrt{21}}{21} \tilde{H}_{\pi}^{21}$
	$ 405_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$	$\frac{2\sqrt{14}}{21} H_{\pi}^{11} - \frac{5\sqrt{7}}{42} \tilde{H}_{\pi}^{11}$	$\frac{3\sqrt{7}}{14} \tilde{H}_{\rho}^{11}$	$\frac{3\sqrt{7}}{14} \tilde{H}_{\pi}^{01}$	$\frac{3\sqrt{7}}{14} \tilde{H}_{\pi}^{21}$
	$ 1_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$	$-\frac{\sqrt{15}}{30} H_{\pi}^{11} - \frac{\sqrt{30}}{15} \tilde{H}_{\pi}^{11}$	$-\frac{\sqrt{15}}{6} H_{\rho}^{11} - \frac{\sqrt{30}}{15} \tilde{H}_{\rho}^{11}$	$-\frac{\sqrt{15}}{6} H_{\pi}^{01} - \frac{\sqrt{30}}{15} \tilde{H}_{\pi}^{01}$	$-\frac{\sqrt{15}}{6} H_{\pi}^{21} - \frac{\sqrt{30}}{15} \tilde{H}_{\pi}^{21}$
	$ 189_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$	$\frac{2\sqrt{10}}{15} H_{\pi}^{11} + \frac{\sqrt{5}}{30} \tilde{H}_{\pi}^{11}$	$-\frac{3\sqrt{5}}{10} \tilde{H}_{\rho}^{11}$	$-\frac{3\sqrt{5}}{10} \tilde{H}_{\pi}^{01}$	$-\frac{3\sqrt{5}}{10} \tilde{H}_{\pi}^{21}$
$1^-(1^{--}) \{^3P_1\}$	Type II	$\chi_{c1}\pi$	$h_c\rho$	$\eta_c\pi \{^1P_1\}$	$\eta_{c2}\pi \{^5P_1\}$
	$ 35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$	$-\frac{1}{6} H_{\pi}^{11} - \frac{\sqrt{2}}{3} \tilde{H}_{\pi}^{11}$	0	0	0
	$ 35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$	$-\frac{1}{6} H_{\pi}^{11} - \frac{\sqrt{2}}{3} \tilde{H}_{\pi}^{11}$	0	0	0
	$ 280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$	$\frac{\sqrt{2}}{3} H_{\pi}^{11} - \frac{1}{6} \tilde{H}_{\pi}^{11}$	0	0	0
	$ 280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$	$\frac{\sqrt{2}}{3} H_{\pi}^{11} - \frac{1}{6} \tilde{H}_{\pi}^{11}$	0	0	0
$1^-(1^{--}) \{^5P_1\}$	Type II	$\chi_{c1}\pi$	$h_c\rho$	$\eta_c\pi \{^1P_1\}$	$\eta_{c2}\pi \{^5P_1\}$
	$ 405_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$	$-\frac{\sqrt{10}}{6} H_{\pi}^{11} + \frac{\sqrt{5}}{6} \tilde{H}_{\pi}^{11}$	0	0	0
	$ 189_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$	$-\frac{\sqrt{5}}{6} H_{\pi}^{11} + \frac{\sqrt{10}}{6} \tilde{H}_{\pi}^{11}$	0	0	0
$0^+(1^{--})$	Type I	$\chi_{c1}\eta$	$h_c\omega$	$\eta_c\eta \{^1P_1\}$	$\eta_{c2}\eta \{^5P_1\}$
	$ 35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$	$\frac{\sqrt{2}}{2} H_{\eta}^{11}$	$-\frac{\sqrt{2}}{2} H_{\omega}^{11}$	$-\frac{\sqrt{2}}{2} H_{\eta}^{01}$	$-\frac{\sqrt{2}}{2} H_{\eta}^{21}$
	$ 35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$	$\frac{\sqrt{2}}{2} H_{\eta}^{11}$	$-\frac{\sqrt{2}}{2} H_{\omega}^{11}$	$-\frac{\sqrt{2}}{2} H_{\eta}^{01}$	$-\frac{\sqrt{2}}{2} H_{\eta}^{21}$
	$ 35_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$	$\frac{\sqrt{3}}{3} H_{\eta}^{11} + \frac{\sqrt{6}}{6} \tilde{H}_{\eta}^{11}$	$\frac{\sqrt{3}}{3} H_{\omega}^{11} + \frac{\sqrt{6}}{6} \tilde{H}_{\omega}^{11}$	$\frac{\sqrt{3}}{3} H_{\eta}^{01} + \frac{\sqrt{6}}{6} \tilde{H}_{\eta}^{01}$	$\frac{\sqrt{3}}{3} H_{\eta}^{21} + \frac{\sqrt{6}}{6} \tilde{H}_{\eta}^{21}$
	$ 35_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$	$\frac{\sqrt{6}}{6} H_{\eta}^{11} - \frac{\sqrt{3}}{3} \tilde{H}_{\eta}^{11}$	$\frac{\sqrt{6}}{6} H_{\omega}^{11} - \frac{\sqrt{3}}{3} \tilde{H}_{\omega}^{11}$	$\frac{\sqrt{6}}{6} H_{\eta}^{01} - \frac{\sqrt{3}}{3} \tilde{H}_{\eta}^{01}$	$\frac{\sqrt{6}}{6} H_{\eta}^{21} - \frac{\sqrt{3}}{3} \tilde{H}_{\eta}^{21}$
	$ 280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$	$-\frac{\sqrt{2}}{2} \tilde{H}_{\eta}^{11}$	$\frac{\sqrt{2}}{2} \tilde{H}_{\omega}^{11}$	$\frac{\sqrt{2}}{2} \tilde{H}_{\eta}^{01}$	$\frac{\sqrt{2}}{2} \tilde{H}_{\eta}^{21}$
$0^+(1^{--}) \{^3P_1\}$	Type II	$\chi_{c1}\eta$	$h_c\omega$	$\eta_c\eta \{^1P_1\}$	$\eta_{c2}\eta \{^5P_1\}$
	$ 1_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$	$\frac{\sqrt{21}}{42} H_{\eta}^{11} + \frac{\sqrt{21}}{42} \tilde{H}_{\eta}^{11}$	$\frac{\sqrt{21}}{6} H_{\omega}^{11} + \frac{\sqrt{21}}{21} \tilde{H}_{\omega}^{11}$	$\frac{\sqrt{21}}{6} H_{\eta}^{01} + \frac{\sqrt{21}}{21} \tilde{H}_{\eta}^{01}$	$\frac{\sqrt{21}}{6} H_{\eta}^{21} + \frac{\sqrt{21}}{21} \tilde{H}_{\eta}^{21}$
	$ 405_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$	$\frac{2\sqrt{14}}{21} H_{\eta}^{11} - \frac{5\sqrt{7}}{42} \tilde{H}_{\eta}^{11}$	$\frac{3\sqrt{7}}{14} \tilde{H}_{\omega}^{11}$	$\frac{3\sqrt{7}}{14} \tilde{H}_{\eta}^{01}$	$\frac{3\sqrt{7}}{14} \tilde{H}_{\eta}^{21}$
	$ 1_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$	$-\frac{\sqrt{15}}{30} H_{\eta}^{11} - \frac{\sqrt{30}}{15} \tilde{H}_{\eta}^{11}$	$-\frac{\sqrt{15}}{6} H_{\omega}^{11} - \frac{\sqrt{30}}{15} \tilde{H}_{\omega}^{11}$	$-\frac{\sqrt{15}}{6} H_{\eta}^{01} - \frac{\sqrt{30}}{15} \tilde{H}_{\eta}^{01}$	$-\frac{\sqrt{15}}{6} H_{\eta}^{21} - \frac{\sqrt{30}}{15} \tilde{H}_{\eta}^{21}$
	$ 189_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$	$\frac{2\sqrt{10}}{15} H_{\eta}^{11} + \frac{\sqrt{5}}{30} \tilde{H}_{\eta}^{11}$	$-\frac{3\sqrt{5}}{10} \tilde{H}_{\omega}^{11}$	$-\frac{3\sqrt{5}}{10} \tilde{H}_{\eta}^{01}$	$-\frac{3\sqrt{5}}{10} \tilde{H}_{\eta}^{21}$
$0^+(1^{--}) \{^5P_1\}$	Type II	$\chi_{c1}\eta$	$h_c\omega$	$\eta_c\eta \{^1P_1\}$	$\eta_{c2}\eta \{^5P_1\}$
	$ 35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$	$-\frac{1}{6} H_{\eta}^{11} - \frac{\sqrt{2}}{3} \tilde{H}_{\eta}^{11}$	0	0	0
	$ 35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$	$-\frac{1}{6} H_{\eta}^{11} - \frac{\sqrt{2}}{3} \tilde{H}_{\eta}^{11}$	0	0	0
	$ 280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$	$\frac{\sqrt{2}}{3} H_{\eta}^{11} - \frac{1}{6} \tilde{H}_{\eta}^{11}$	0	0	0
	$ 280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$	$\frac{\sqrt{2}}{3} H_{\eta}^{11} - \frac{1}{6} \tilde{H}_{\eta}^{11}$	0	0	0

There are two types of  $1^{--}$  states. For the isovector  $1^{--}$  states of type I, the allowed S-wave decay modes are  $\chi_{c1}\pi$  and  $h_c\rho$ , where the spin configurations  $(1_H \otimes 1_L)_0^{--}$  and  $(0_H \otimes 1_L)_0^{--}$  are dominant respectively. The allowed P-wave decay modes are  $\eta_c\pi$  and  $\eta_{c2}\pi$ , where the spin configuration  $(0_H \otimes 1_L)_0^{--}$  is dominant. The allowed decay modes from  $|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)}$  and  $|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)}$  are governed by the color singlet, while the decay modes from  $|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)}$  and  $|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)}$  are governed by the color octet. The mode  $\chi_{c1}\pi$  is allowed for the isovector  $1^{--} \{^1P_1\}$ ,  $1^{--} \{^3P_1\}$  and  $1^{--} \{^5P_1\}$  states of type II.

For the  $1^{-+} \{^1P_1\}$ ,  $1^{-+} \{^3P_1\}$  and  $1^{-+} \{^5P_1\}$  states of type II, the  $1^{-+} \{^1P_1\}$  can decay into  $h_c\rho$ ,  $\eta_c\pi \{^1P_1\}$  and  $\eta_{c2}\pi \{^5P_1\}$ . These decay modes are suppressed for the states  $1^{-+} \{^3P_1\}$  and  $1^{-+} \{^5P_1\}$ . The re-coupled color-spin wave functions of the  $1^{-+} \{^3P_1\}$  and  $1^{-+} \{^5P_1\}$  states of type II do not contain the spin configuration  $(0_H \otimes 1_l)_0^{+-}$ . Therefore, their decays into  $h_c\rho$ ,  $\eta_c\pi \{^1P_1\}$  and  $\eta_{c2}\pi \{^5P_1\}$  are strongly suppressed in the heavy quark limit. The decay modes  $h_c\rho$ ,  $\eta_c\pi \{^1P_1\}$  and  $\eta_{c2}\pi \{^5P_1\}$  of the  $1^{-}(1^{-+}) \{^1P_1\}$  states  $|405_{cs}, 1_c, 1\rangle_{(21,21)}$  and  $|189_{cs}, 1_c, 1\rangle_{(15,15)}$  of type II are governed by their color octet and the spin configuration  $(0_H \otimes 1_l)_0^{+-}$ .

The isoscalar  $1^{-+}$  states of type I can decay into  $\chi_{c1}\eta$ ,  $h_c\omega$ ,  $\eta_c\eta \{^1P_1\}$  and  $\eta_{c2}\eta \{^5P_1\}$ . These decay modes are also allowed for the  $1^{-+} \{^1P_1\}$  states of Type II. However, the states  $1^{-+} \{^3P_1\}$  and  $1^{-+} \{^5P_1\}$  of type II do not decay into  $h_c\omega$ ,  $\eta_c\eta \{^1P_1\}$  and  $\eta_{c2}\eta \{^5P_1\}$ . These decay modes are dominated by the spin configuration  $(0_H \otimes 1_l)_0^{+-}$  which does not appear in the states  $1^{-+} \{^3P_1\}$  and  $1^{-+} \{^5P_1\}$  of type II. The decay mode  $\chi_{c1}\eta$  is allowed for all the isoscalar  $1^{-+}$  states.

TABLE VI: The decay matrix elements of the tetraquarks with  $J^{PC} = 0^{-+}, 0^{-+}$ . The reduced matrix elements  $H_{\alpha}^{ij} \propto \langle Q, i || H_{eff}(\alpha) || j \rangle$ , where the indices  $i$  and  $j$  denote the light spin of the final and initial hadron respectively, and  $Q$  is the angular momentum of the final light meson. The quantum numbers in the bracelets represent the total angular momentum configurations of the final state particles.

$I^G(J^{PC})$	Initial state	Final state	$I^G(J^{PC})$	Initial state	Final state
$1^{-}(0^{-+})$	Type I	$\chi_{c0}\pi$	$1^{+}(0^{-+})$	Type II	$\chi_{c1}\rho$
	$ 35_{cs}, 1_c, 0\rangle_{(21,21)}$	$\frac{\sqrt{2}}{2} H_{\pi}^{11}$		$ 35_{cs}, 1_c, 0\rangle_{(21,21)}$	$\frac{\sqrt{2}}{2} H_{\rho}^{11}$
	$ 35_{cs}, 1_c, 0\rangle_{(15,15)}$	$\frac{\sqrt{2}}{2} H_{\pi}^{11}$		$ 35_{cs}, 1_c, 0\rangle_{(15,15)}$	$\frac{\sqrt{2}}{2} H_{\rho}^{11}$
	$ 35_{cs}, 1_c, 0\rangle_{(21,15)}$	$\frac{\sqrt{3}}{3} H_{\pi}^{11} + \frac{\sqrt{6}}{6} \tilde{H}_{\pi}^{11}$		$ 35_{cs}, 1_c, 0\rangle_{(21,15)}$	$\frac{\sqrt{3}}{3} H_{\rho}^{11} + \frac{\sqrt{6}}{6} \tilde{H}_{\rho}^{11}$
	$ 35_{cs}, 1_c, 0\rangle_{(15,21)}$	$\frac{\sqrt{6}}{6} H_{\pi}^{11} - \frac{\sqrt{3}}{3} \tilde{H}_{\pi}^{11}$		$ 35_{cs}, 1_c, 0\rangle_{(15,21)}$	$\frac{\sqrt{6}}{6} H_{\rho}^{11} - \frac{\sqrt{3}}{3} \tilde{H}_{\rho}^{11}$
	$ 280_{cs}, 1_c, 0\rangle_{(21,15)}$	$-\frac{\sqrt{2}}{2} \tilde{H}_{\pi}^{11}$		$ 280_{cs}, 1_c, 0\rangle_{(21,15)}$	$-\frac{\sqrt{2}}{2} \tilde{H}_{\rho}^{11}$
	$ 280_{cs}, 1_c, 0\rangle_{(15,21)}$	$-\frac{\sqrt{2}}{2} \tilde{H}_{\pi}^{11}$		$ 280_{cs}, 1_c, 0\rangle_{(15,21)}$	$-\frac{\sqrt{2}}{2} \tilde{H}_{\rho}^{11}$
$1^{-}(0^{-+})$	Type II	$\chi_{c0}\pi$	$1^{+}(0^{-+})$	Type I	$\chi_{c1}\rho$
	$ 35_{cs}, 1_c, 0\rangle_{(21,21)}$	$-\frac{1}{3} H_{\pi}^{11} - \frac{2\sqrt{2}}{3} \tilde{H}_{\pi}^{11}$		$ 35_{cs}, 1_c, 0\rangle_{(21,21)}$	$-\frac{1}{3} H_{\rho}^{11} - \frac{2\sqrt{2}}{3} \tilde{H}_{\rho}^{11}$
	$ 35_{cs}, 1_c, 0\rangle_{(15,15)}$	$-\frac{1}{3} H_{\pi}^{11} - \frac{2\sqrt{2}}{3} \tilde{H}_{\pi}^{11}$		$ 35_{cs}, 1_c, 0\rangle_{(15,15)}$	$-\frac{1}{3} H_{\rho}^{11} - \frac{2\sqrt{2}}{3} \tilde{H}_{\rho}^{11}$
	$ 280_{cs}, 1_c, 0\rangle_{(21,15)}$	$\frac{2\sqrt{2}}{3} H_{\pi}^{11} - \frac{1}{3} \tilde{H}_{\pi}^{11}$		$ 280_{cs}, 1_c, 0\rangle_{(21,15)}$	$\frac{2\sqrt{2}}{3} H_{\rho}^{11} - \frac{1}{3} \tilde{H}_{\rho}^{11}$
	$ 280_{cs}, 1_c, 0\rangle_{(15,21)}$	$\frac{2\sqrt{2}}{3} H_{\pi}^{11} - \frac{1}{3} \tilde{H}_{\pi}^{11}$		$ 280_{cs}, 1_c, 0\rangle_{(15,21)}$	$\frac{2\sqrt{2}}{3} H_{\rho}^{11} - \frac{1}{3} \tilde{H}_{\rho}^{11}$
$0^{+}(0^{-+})$	Type I	$\chi_{c0}\eta$	$0^{-}(0^{-+})$	Type II	$\chi_{c1}\omega$
	$ 35_{cs}, 1_c, 0\rangle_{(21,21)}$	$\frac{\sqrt{2}}{2} H_{\eta}^{11}$		$ 35_{cs}, 1_c, 0\rangle_{(21,21)}$	$\frac{\sqrt{2}}{2} H_{\omega}^{11}$
	$ 35_{cs}, 1_c, 0\rangle_{(15,15)}$	$\frac{\sqrt{2}}{2} H_{\eta}^{11}$		$ 35_{cs}, 1_c, 0\rangle_{(15,15)}$	$\frac{\sqrt{2}}{2} H_{\omega}^{11}$
	$ 35_{cs}, 1_c, 0\rangle_{(21,15)}$	$\frac{\sqrt{3}}{3} H_{\eta}^{11} + \frac{\sqrt{6}}{6} \tilde{H}_{\eta}^{11}$		$ 35_{cs}, 1_c, 0\rangle_{(21,15)}$	$\frac{\sqrt{3}}{3} H_{\omega}^{11} + \frac{\sqrt{6}}{6} \tilde{H}_{\omega}^{11}$
	$ 35_{cs}, 1_c, 0\rangle_{(15,21)}$	$\frac{\sqrt{6}}{6} H_{\eta}^{11} - \frac{\sqrt{3}}{3} \tilde{H}_{\eta}^{11}$		$ 35_{cs}, 1_c, 0\rangle_{(15,21)}$	$\frac{\sqrt{6}}{6} H_{\omega}^{11} - \frac{\sqrt{3}}{3} \tilde{H}_{\omega}^{11}$
	$ 280_{cs}, 1_c, 0\rangle_{(21,15)}$	$-\frac{\sqrt{2}}{2} \tilde{H}_{\eta}^{11}$		$ 280_{cs}, 1_c, 0\rangle_{(21,15)}$	$-\frac{\sqrt{2}}{2} \tilde{H}_{\omega}^{11}$
	$ 280_{cs}, 1_c, 0\rangle_{(15,21)}$	$-\frac{\sqrt{2}}{2} \tilde{H}_{\eta}^{11}$		$ 280_{cs}, 1_c, 0\rangle_{(15,21)}$	$-\frac{\sqrt{2}}{2} \tilde{H}_{\omega}^{11}$
$0^{+}(0^{-+})$	Type II	$\chi_{c0}\eta$	$0^{-}(0^{-+})$	Type I	$\chi_{c1}\omega$
	$ 35_{cs}, 1_c, 0\rangle_{(21,21)}$	$-\frac{1}{3} H_{\eta}^{11} - \frac{2\sqrt{2}}{3} \tilde{H}_{\eta}^{11}$		$ 35_{cs}, 1_c, 0\rangle_{(21,21)}$	$-\frac{1}{3} H_{\omega}^{11} - \frac{2\sqrt{2}}{3} \tilde{H}_{\omega}^{11}$
	$ 35_{cs}, 1_c, 0\rangle_{(15,15)}$	$-\frac{1}{3} H_{\eta}^{11} - \frac{2\sqrt{2}}{3} \tilde{H}_{\eta}^{11}$		$ 35_{cs}, 1_c, 0\rangle_{(15,15)}$	$-\frac{1}{3} H_{\omega}^{11} - \frac{2\sqrt{2}}{3} \tilde{H}_{\omega}^{11}$
	$ 280_{cs}, 1_c, 0\rangle_{(21,15)}$	$\frac{2\sqrt{2}}{3} H_{\eta}^{11} - \frac{1}{3} \tilde{H}_{\eta}^{11}$		$ 280_{cs}, 1_c, 0\rangle_{(21,15)}$	$\frac{2\sqrt{2}}{3} H_{\omega}^{11} - \frac{1}{3} \tilde{H}_{\omega}^{11}$
	$ 280_{cs}, 1_c, 0\rangle_{(15,21)}$	$\frac{2\sqrt{2}}{3} H_{\eta}^{11} - \frac{1}{3} \tilde{H}_{\eta}^{11}$		$ 280_{cs}, 1_c, 0\rangle_{(15,21)}$	$\frac{2\sqrt{2}}{3} H_{\omega}^{11} - \frac{1}{3} \tilde{H}_{\omega}^{11}$

Both of the  $0^{-+}$  and the  $0^{-+}$  tetraquark states have two types. In Table VI, we notice that all the isovector  $0^{-+}$  states can decay into  $\chi_{c0}\pi$ , which is governed by the spin configuration  $(1_H \otimes 1_l)_0^{+-}$ . The difference between the  $1^{-}(0^{-+})$  states of type I and the  $1^{-}(0^{-+})$  states of type II is reflected in their decay modes. The decay modes  $h_c\rho$  and  $\eta_{c0}(980)$  are dominated by the spin configuration  $(0_H \otimes 1_l)_0^{+-}$ , which exists in the  $1^{-}(0^{-+})$  states of type I and is absent in the  $1^{-}(0^{-+})$  states of type II. Thus, these decay modes from the  $1^{-}(0^{-+})$  states of type II are suppressed.

The same situation occurs for the isoscalar  $0^{-+}$  states of type I and type II. The decay modes  $h_c\omega$  and  $\eta_c\sigma$  are governed by the spin configuration  $(0_H \otimes 1_l)_0^{+-}$  as well. Nevertheless, this spin configuration  $(0_H \otimes 1_l)_0^{+-}$  is absent in the  $0^{+}(0^{-+})$  states of type II. The heavy spin and light spin are not conserved for the decay modes  $h_c\omega$  and  $\eta_c\sigma$ . These decay widths are strongly suppressed in the heavy quark limit. The decay modes  $h_c\omega$  and  $\eta_c\sigma$  are allowed for the  $0^{+}(0^{-+})$  states of type I. Both types of states can decays into  $\chi_{c0}\eta$ .

The  $\chi_{c1}\rho$ ,  $h_{c0}(980) \{^3P_0\}$ ,  $\eta_c\rho \{^3P_0\}$  and  $J/\psi\pi \{^3P_0\}$  are the allowed decay modes for the  $1^{+}(0^{-+})$  states of type II. The  $\chi_{c1}\rho$  and  $J/\psi\pi \{^3P_0\}$  modes are governed by the spin configuration  $(1_H \otimes 1_l)_0^{+-}$  while the  $h_{c0}(980) \{^3P_0\}$  and the  $\eta_c\rho \{^3P_0\}$  depend on the spin configuration  $(0_H \otimes 1_l)_0^{+-}$ . As shown in Appendix B, the  $1^{+}(0^{-+})$  states of type I contain the spin configuration  $(1_H \otimes 1_l)_0^{+-}$  only. They can decay into  $\chi_{c1}\rho$  and  $J/\psi\pi \{^3P_0\}$ . But they can not decay into  $h_{c0}(980) \{^3P_0\}$  and  $\eta_c\rho \{^3P_0\}$  in the heavy quark limit.

The  $0^-(0^{--})$  states of type II decay into  $\chi_{c1}\omega$ ,  $h_c\sigma$   $\{^3P_0\}$ ,  $\eta_c\omega$   $\{^3P_0\}$  and  $J/\psi\eta$   $\{^3P_0\}$ . However, the modes  $h_c\sigma$   $\{^3P_0\}$  and  $\eta_c\omega$   $\{^3P_0\}$  of the  $0^-(0^{--})$  states of type I are suppressed.

## V. DECAY PATTERNS OF $Y(4260)$

The charmoniumlike state  $Y(4260)$  with  $I^G(J^{PC}) = 0^-(1^{--})$  was first discovered by the BaBar Collaboration in the  $e^+e^- \rightarrow J/\psi\pi^+\pi^-$  process [73]. There have been extensive experimental and theoretical investigations of this puzzling state [33, 82, 84–99]. Some authors proposed that  $Y(4260)$  could be reproduced by the interference of  $e^+e^- \rightarrow \psi(4160)/\psi(4415) \rightarrow J/\psi\pi^+\pi^-$  and the background contribution [97].  $Y(4260)$  may be an exotic hybrid charmonium state or a conventional charmonium [33]. In this work, we compare the decay patterns under various resonant assumptions.

From the conservation of the total angular momentum, parity, C parity and G parity, the allowed decay modes of  $Y(4260)$  are  $J/\psi\sigma$ ,  $\psi(1^3D_1)\sigma$ ,  $h_c\eta$ ,  $\chi_{cJ}\omega$  ( $J = 0, 1, 2$ ),  $J/\psi\eta$   $\{^3P_1\}$ ,  $\psi(1^3D_1)\eta$   $\{^3P_1\}$  and  $\psi(1^3D_2)\eta$   $\{^5P_1\}$ . We collect the current experimental information on the decay modes of  $Y(4260)$  in Table VII.

TABLE VII: Experimental information on the decay modes of  $Y(4260)$ .

$Y(4260)$ decay modes	Observation results
$J/\psi\pi^+\pi^-$ ; $J/\psi\pi^0\pi^0$ ; $J/\psi K^+K^-$	seen
$X(3872)\gamma$	seen
$J/\psi\eta$ ; $J/\psi\eta'$ ; $J/\psi\eta\eta$ ; $J/\psi\pi^0$ ; $J/\psi\pi^+\pi^-\pi^0$	not seen
$\psi(2S)\pi^+\pi^-$ ; $\psi(2S)\eta$	not seen
$\chi_{c0}\omega$ ; $\chi_{c1}\gamma$ ; $\chi_{c2}\gamma$ ; $\chi_{c1}\pi^+\pi^-\pi^0$ ; $\chi_{c2}\pi^+\pi^-\pi^0$	not seen
$h_c(1P)\pi^+\pi^-$ ; $\phi\pi^+\pi^-$	not seen
$D\bar{D}$ ; $D^*\bar{D} + c.c.$ ; $D^*\bar{D}^*$	not seen
$D^*\bar{D}^*\pi$ ; $D^0D^{*-} + c.c.$ ; $D\bar{D}^* + c.c.$	not seen
$D_s^+D_s^-$ ; $D_s^{*+}D_s^- + c.c.$ ; $D_s^{*+}D_s^{*-}$	not seen
$J/\psi\eta\pi^0$	not seen

### A. Decay patterns of $Y(4260)$ as a conventional charmonium

$Y(4260)$  could be a conventional charmonium the  $\psi(4S)$  or the mixture of 4S and 3D vector charmonia  $Y(4260)$  [33, 84, 85, 87]. Recall that the spin structures of  $\psi(4S)$  and  $\psi(3D)$  are  $(1_H \otimes 0_I)_1^{--}$  and  $(1_H \otimes 2_I)_1^{--}$  respectively. If  $Y(4260)$  is the mixture of  $\psi(4S)$  and  $\psi(3D)$ , it can easily decay into  $J/\psi\sigma$ ,  $\psi(1^3D_1)\sigma$  and  $J/\psi f_0(980)$ . However, the strong decay mode  $h_c\eta$  and radiative decay modes  $\eta_c\gamma(M1)$  and  $\eta_{c2}\gamma(M1)$  are all dominated by the spin configuration  $(0_H \otimes 1_I)_1^{--}$  and are suppressed in this case.

For comparison, the spin configuration  $(0_H \otimes 1_I)_1^{--}$  appears in the color-spin wave functions of the  $0^-(1^{--})$   $\{^1P_1\}$  tetraquarks of type I. All the  $h_c\eta$  and  $\eta_c\gamma(M1)$  and  $\eta_{c2}\gamma(M1)$  modes are allowed if  $Y(4260)$  is the  $0^-(1^{--})$   $\{^1P_1\}$  of type I. In other words, the strong decay mode  $h_c\eta$  and radiative decay modes  $\eta_c\gamma(M1)$  and  $\eta_{c2}\gamma(M1)$  can provide a way to distinguish the tetraquark assumptions of  $Y(4260)$  from the conventional charmonia assumptions.

### B. Decay patterns of $Y(4260)$ as a Molecule

We made a comprehensive investigation on the decay patterns  $Y(4260)$  as a molecular state with  $I^G(J^{PC}) = 0^-(1^{--})$  in Refs. [67, 68]. With the molecular assumptions,  $Y(4260)$  can decay into  $h_c\eta$ ,  $\chi_{cJ}\omega$  ( $J = 0, 1, 2$ ),  $J/\psi\eta$   $\{^3P_1\}$ ,  $\psi(1^3D_1)\eta$   $\{^3P_1\}$  and  $\psi(1^3D_2)\eta$   $\{^5P_1\}$ . However, the decay modes  $J/\psi\sigma$ ,  $\psi(1^3D_1)\sigma$  and  $J/\psi f_0(980)$  are strongly suppressed under the heavy quark symmetry. Within the molecular scheme,  $Y(4260)$  may be the mixture of the pure  $D_1\bar{D}$  and  $D_1'\bar{D}$ . Considering this mixing effect,  $J/\psi\sigma$ ,  $\psi(1^3D_1)\sigma$  and  $J/\psi f_0(980)$  are also suppressed.

$Y(4260)$  was first discovered in the  $J/\psi\pi^+\pi^-$  final states where the  $\pi^+\pi^-$  pair is in the S-wave, which may arise from the intermediate  $\sigma$  and  $f_0(980)$  resonances. The molecular scheme is unable to explain the discovery mode  $J/\psi\pi^+\pi^-$  of  $Y(4260)$ .

As the isoscalar  $1^{--}$  molecule,  $Y(4260)$  is composed of  $D_0\bar{D}^*$ ,  $D'_1\bar{D}$ ,  $D_1\bar{D}$ ,  $D'_1\bar{D}^*$ ,  $D_1\bar{D}^*$  or  $D_2\bar{D}^*$ . All these molecules are dominated by the spin configurations  $(0_H \otimes 1_l)_1^{--}$  and  $(1_H \otimes 1_l)_1^{--}$ . The same spin configurations also appear in the open charm final states such as  $D\bar{D}$ ,  $D^*\bar{D}$ ,  $D^*\bar{D}^*$ . In other words,  $Y(4260)$  can decay into the open-charm modes easily if it's a molecular state.

In Table VII, we notice that the  $J/\psi\eta$  and  $J/\psi\pi^+\pi^-\pi^0$  are "not seen" experimentally. These final states are governed by the spin configuration  $(1_H \otimes 1_l)_1^{--}$ , which exists in the  $1^{--}$  molecular structures. Thus these decays should be allowed for the molecule assumption.

Moreover, the strong decay mode  $h_c\eta$  and radiative decay modes  $\eta_c\gamma(M1)$  and  $\eta_{c2}\gamma(M1)$  are allowed for the molecule scheme.

The discovery mode  $J/\psi\pi^+\pi^-$  and non-observation of the open-charm decay modes  $D\bar{D}$ ,  $D^*\bar{D}$ ,  $D^*\bar{D}^*$  and hidden-charm decay modes  $J/\psi\eta$  and  $J/\psi\pi^+\pi^-\pi^0$  disfavor the molecule assumption of  $Y(4260)$ .

### C. Decay patterns of $Y(4260)$ as a tetraquark state

There are two types of isoscalar  $1^{--}$  tetraquarks. For "type I", the P-wave excitation exists between the diquark and anti-diquark pair. For "type II", the P-wave excitation exists inside either the diquark or anti-diquark.

The total spin of the four quarks can be 0, 1 and 2. The isoscalar  $1^{--}$  tetraquarks of type I can be further categorized into three kinds of structures,  $0^-(1^{--}) \{^1P_1\}$ ,  $0^-(1^{--}) \{^3P_1\}$  and  $0^-(1^{--}) \{^5P_1\}$ .

According to Table IV, all the isoscalar  $1^{--}$  tetraquark states of type I can easily decay into  $J/\psi\sigma$ ,  $\psi(1^3D_1)\sigma$  and  $J/\psi f_0(980)$ . The  $J/\psi\pi^+\pi^-$  mode are allowed for all the isoscalar  $1^{--}$  tetraquarks of type I.

Nevertheless, the isoscalar  $1^{--}$  tetraquarks of type II do not decay into  $J/\psi\sigma$ ,  $\psi(1^3D_1)\sigma$  and  $J/\psi f_0(980)$  in the heavy quark symmetry limit. If  $Y(4260)$  turns out be a tetraquark state, its P-wave excitation exists between the diquark and anti-diquark pair.

From Table IV, we notice that the decay mode  $h_c\eta$  is allowed for the  $0^-(1^{--}) \{^1P_1\}$  tetraquarks of type I while  $h_c\eta$  is strongly suppressed for the  $0^-(1^{--}) \{^3P_1\}$  and  $0^-(1^{--}) \{^5P_1\}$  tetraquarks of type I. The  $h_c\eta$  decay mode provides a way to distinguish the  $0^-(1^{--}) \{^1P_1\}$  tetraquark state from  $0^-(1^{--}) \{^3P_1\}$  and  $0^-(1^{--}) \{^5P_1\}$ .

The decay mode  $\chi_{c0}\omega$  is also allowed if  $Y(4260)$  is a  $0^-(1^{--})$  tetraquark of type I. Unfortunately, the decay phase space is tiny. Hence the experimental measurement of the  $\chi_{c0}\omega$  mode will be difficult.

The P-wave open charm modes  $D\bar{D}$ ,  $D^*\bar{D}$ ,  $D^*\bar{D}^*$  are dominated by the spin configurations  $(0_H \otimes 1_l)_1^{--}$ ,  $(1_H \otimes 0_l)_1^{--}$ ,  $(1_H \otimes 1_l)_1^{--}$  and  $(1_H \otimes 2_l)_1^{--}$ . The S-wave open charm final states  $D_0\bar{D}^*$ ,  $D'_1\bar{D}$ ,  $D_1\bar{D}$ ,  $D'_1\bar{D}^*$ ,  $D_1\bar{D}^*$  or  $D_2\bar{D}^*$  are governed by the spin configurations  $(0_H \otimes 1_l)_1^{--}$  and  $(1_H \otimes 1_l)_1^{--}$ . The open charm final states  $D^*\bar{D}\pi$ ,  $D^*\bar{D}^*\pi$  also contain the  $(0_H \otimes 1_l)_1^{--}$ ,  $(1_H \otimes 0_l)_1^{--}$ ,  $(1_H \otimes 1_l)_1^{--}$  and  $(1_H \otimes 2_l)_1^{--}$  configurations. The isoscalar  $1^{--}$  tetraquark states of type I contain the spin configuration  $(1_H \otimes 1_l)_1^{--}$ . Therefore, all the above open charm decay modes are allowed under the heavy quark symmetry. The non-observation of the open charm decay modes remains a big challenge to the tetraquark assumption of  $Y(4260)$ .

### D. Decay patterns of $Y(4260)$ as a hybrid Charmonium

#### 1. Scenario A

$Y(4260)$  was proposed as a good candidate of the hybrid charmonium [33]. With Scenario A, we assume that  $Y(4260)$  is composed of a pair of  $c\bar{c}$  and a gluon. We use  $l_g = 0, 1$  to denote the E1 and M1 gluon in the hybrid charmonium respectively.  $l_{c\bar{c}}$  is the orbital momentum of the  $c\bar{c}$  pair, and  $s_{c\bar{c}}$  is the spin of  $c\bar{c}$ . Thus, we have  $(l_g, l_{c\bar{c}}, s_{c\bar{c}}) = (0, 1, 1)$  or  $(l_g, l_{c\bar{c}}, s_{c\bar{c}}) = (1, 0, 0)$ . The  $s_{c\bar{c}} = 1$  and  $l_{c\bar{c}} = 1$  in the  $(0, 1, 1)$  can couple into 0, 1, 2. We use the superscripts 0, 1, 2 to denote the three states. The spin structures of  $Y(4260)$  read

$$Y(4260)_{(0,1,1)}^0 = \frac{1}{3}(1_H \otimes 0_l)_1^{--} - \frac{\sqrt{3}}{3}(1_H \otimes 1_l)_1^{--} + \frac{\sqrt{5}}{3}(1_H \otimes 2_l)_1^{--}, \quad (28)$$

$$Y(4260)_{(0,1,1)}^1 = -\frac{\sqrt{3}}{3}(1_H \otimes 0_l)_1^{--} + \frac{1}{2}(1_H \otimes 1_l)_1^{--} + \frac{\sqrt{15}}{6}(1_H \otimes 2_l)_1^{--}, \quad (29)$$

$$Y(4260)_{(0,1,1)}^2 = \frac{\sqrt{5}}{3}(1_H \otimes 0_l)_1^{--} - \frac{\sqrt{15}}{6}(1_H \otimes 1_l)_1^{--} + \frac{1}{6}(1_H \otimes 2_l)_1^{--}, \quad (30)$$

$$Y(4260)_{(1,0,0)} = (0_H \otimes 1_l)_1^{--}. \quad (31)$$

We notice that the states  $Y(4260)_{(0,1,1)}^0$ ,  $Y(4260)_{(0,1,1)}^1$  and  $Y(4260)_{(0,1,1)}^2$  contain the spin configuration  $(1_H \otimes 0_l)_1^{--}$ , while the state  $Y(4260)_{(1,0,0)}$  contain the spin configuration  $(0_H \otimes 1_l)_1^{--}$  only. The discovery mode  $J/\psi\pi^+\pi^-$  is governed by the spin configuration  $(1_H \otimes 0_l)_1^{--}$ . Thus,  $Y(4260)_{(1,0,0)}$  is disfavored under heavy quark symmetry.

The P-wave decay modes  $D\bar{D}$ ,  $D^*\bar{D} + c.c.$ ,  $D^*\bar{D}^* D_s^+ D_s^-$ ,  $D_s^{*+} D_s^- + c.c.$ ,  $D_s^{*+} D_s^{*-}$  and  $D^*\bar{D}^* \pi$  are dominated by the spin configurations  $(0_H \otimes 1_l)_1^{--}$ ,  $(1_H \otimes 0_l)_1^{--}$ ,  $(1_H \otimes 1_l)_1^{--}$  and  $(1_H \otimes 2_l)_1^{--}$ . The S-wave open charm modes which contain a P-wave charmed meson such as  $D_0\bar{D}^*$ ,  $D_1'\bar{D}$ ,  $D_1\bar{D}$ ,  $D_1'\bar{D}^*$ ,  $D_1\bar{D}^*$  or  $D_2\bar{D}^*$  are governed by the spin configurations  $(0_H \otimes 1_l)_1^{--}$  and  $(1_H \otimes 1_l)_1^{--}$ . According to the spin structures listed in Eqs. (28)-(31),  $Y(4260)$  with the structure  $(0, 1, 1)$  should decay into the above open charm modes easily. Such a hybrid charmonium is composed of a  $c\bar{c}$  pair and one M1 color-magnetic gluon.

We notice that the hybrid charmonium and hidden-charm tetraquarks have very similar strong decay patterns. The underlying dynamics is quite transparent. The gluon within the hybrid charmonium can easily fluctuate into a  $q\bar{q}$  light quark pair to form the hidden-charm tetraquark states.

## 2. Scenario B

When the hybrid charmonium decays, the gluon may pull one or more soft gluons from the  $c\bar{c}$  pair. Now the vector hybrid charmonium is effectively composed of two gluons with  $J^{PC} = 0^{++}$  and a  $J^{PC} = 1^{--}$   $c\bar{c}$  pair with  $l_{c\bar{c}} = 0$ ,  $s_{c\bar{c}} = 1$  and  $l_g = 0$ . Under this specific assumption,  $Y(4260)$  have the spin configuration  $(1_H \otimes 0_l)_1^{--}$  only and decays into  $J/\psi \pi^+ \pi^-$  easily under heavy quark symmetry.  $Y(4260)$  also decays into the P-wave open charm modes such as  $D\bar{D}$ ,  $D^*\bar{D} + c.c.$ ,  $D^*\bar{D}^* D_s^+ D_s^-$ ,  $D_s^{*+} D_s^- + c.c.$ ,  $D_s^{*+} D_s^{*-}$ . However,  $Y(4260)$  does not decay into the open charm modes  $D_0\bar{D}^*$ ,  $D_1'\bar{D}$ ,  $D_1\bar{D}$ ,  $D_1'\bar{D}^*$ ,  $D_1\bar{D}^*$  and  $D_2\bar{D}^*$  since they are dominated by the spin configurations  $(0_H \otimes 1_l)_1^{--}$  and  $(1_H \otimes 1_l)_1^{--}$ .

For both Scenario A and B,  $Y(4260)$  will decay into  $\chi_{c0}\omega$ ,  $\chi_{c1}\gamma$ ,  $\chi_{c2}\gamma$ ,  $\chi_{c1}\pi^+\pi^-\pi^0$  and  $\chi_{c2}\pi^+\pi^-\pi^0$  under heavy quark symmetry since these modes are governed by the spin configurations  $(1_H \otimes 0_l)_1^{--}$ ,  $(1_H \otimes 1_l)_1^{--}$  and  $(1_H \otimes 2_l)_1^{--}$ . The experimental measurement of the radiative decay modes  $\chi_{c1}\gamma$ ,  $\chi_{c2}\gamma$  will be desirable. For both scenarios, the open charm decay modes of  $Y(4260)$  are allowed in the heavy quark symmetry limit. The non-observation of the open charm decay modes is also very puzzling within the hybrid charmonium framework.

## VI. DECAY PATTERNS OF $X(3872)$ , $Z_c(3900)$ , $Z_c(4025)$ , $Y(4360)$ AND $Z_c(4200)$

### A. $X(3872)$

The charmoniumlike state  $X(3872)$  was first observed by the Belle Collaboration in the  $J/\psi \pi^+ \pi^-$  invariant mass spectrum of  $B \rightarrow K J/\psi \pi^+ \pi^-$  [40]. There exist extensive discussions of the underlying structure of  $X(3872)$ . Some plausible interpretations include the  $D\bar{D}^*$  molecular state with  $J^{PC} = 1^{++}$ , the conventional charmonium  $\chi'_{c1}$  or a mixture of  $c\bar{c}$  and  $D\bar{D}^{*st}$ .

The  $1^{++}$  molecular state and  $\chi'_{c1}$  can easily decay into the S-wave decay mode  $\chi_{c1}\sigma$  through the tail of the broad  $\sigma$  resonance. The potentially allowed P-wave decay modes  $\chi_{c0}\eta$ ,  $\chi_{c1}\eta$  and  $\chi_{c2}\eta$  are forbidden by kinematics.

As a  $1^{++}$  tetraquark state, the  $\chi_{c1}\sigma$  mode of  $X(3872)$  is suppressed by the transition between the color octet and color singlet which needs the exchange of soft gluons. However, such a suppression may not be very strong. The experimental observation of the  $\chi_{c1}(\pi\pi)_{S\text{-wave}}$  mode will further test the tetraquark assumption of  $X(3872)$ .

### B. $Z_c(3900)$ and $Z_c(4025)$

As the first charged charmonium-like state,  $Z_c(3900)$  was observed in the  $J/\psi \pi^\pm$  invariant mass spectrum of  $e^+e^- \rightarrow J/\psi \pi^+ \pi^-$  by BESIII collaboration [10], and later confirmed by Belle in the same process.  $Z_c(4020)$  and  $Z_c(4025)$  were observed in the  $e^+e^- \rightarrow h_c \pi^+ \pi^-$  and  $e^+e^- \rightarrow (D^*\bar{D}^*)^\pm \pi^\mp$  at  $\sqrt{s} = 4.26$  GeV respectively [9, 10, 10, 11, 11, 12, 15, 16, 18, 19, 60]. We treat these two signals as the same state. Their neutral partners have quantum numbers  $I^G(J^{PC}) = 1^+(1^{+-})$ .

$Z_c(3900)$  and  $Z_c(4020)$  are speculated to be either the isovector  $D^*\bar{D}$  and  $D^*\bar{D}^*$  molecular states with  $I^G(J^P) = 1^+(1^+)$  or the  $1^+$  tetraquark states.

If  $Z_c(3900)$  and  $Z_c(4020)$  are molecular states, their decay modes include  $J/\psi \pi$ ,  $\psi' \pi$ ,  $\eta_c \rho$ ,  $\eta_{c2} \rho$  and also the P-wave decay mode  $h_c \pi$ . However, if  $Z_c(3900)$  and  $Z_c(4020)$  are the  $1^+(1^+)$  tetraquarks, the decay mode  $h_c \pi$  is suppressed in the heavy quark symmetry limit. If they are the  $1^+(1^+)$  tetraquarks with the  $280_{cs}$  color-spin representation,  $h_c \pi$ ,  $J/\psi \pi$ ,  $\psi' \pi$ ,  $\eta_c \rho$  and  $\eta_{c2} \rho$  are all suppressed, which implies that the states  $[280_{cs}, 1_c, 1]_{(21,15)}$  and  $[280_{cs}, 1_c, 1]_{(15,21)}$  with  $I^G(J^P) = 1^+(1^+)$  have rather narrow widths.

No matter whether  $Z_c(3900)$  and  $Z_c(4020)$  are molecules or tetraquarks, their decay into  $\psi(1^3D_1)\pi$  is strongly suppressed due to the non-conservation of the light spin.

Since  $Z_c(3900)$  was observed in the  $J/\psi \pi^\pm$  invariant mass spectrum of  $e^+e^- \rightarrow J/\psi \pi^+ \pi^-$ , its possibility as a  $1^+(1^+)$  tetraquark with the  $280_{cs}$  color-spin representation is relatively small while the possibility as a  $1^+(1^+)$  tetraquark with the  $35_{cs}$  color-spin representation is not excluded.



Whether  $Z_c(3900)$  is a good candidate of the  $1^+(1^+)$  molecular state or a tetraquark state with the  $35_{cs}$  color-spin representation can be judged by the P-wave decay mode  $h_c\pi$ . Since  $Z_c(4020)$  was observed in the  $h_c\pi^\pm$  invariant mass spectrum, the tetraquark assumption of  $Z_c(4020)$  is disfavored under the heavy quark symmetry. It is probably a  $D^*\bar{D}^*$  molecular state with  $I^G(J^P) = 1^+(1^+)$ .

### C. $Y(4360)$

The Belle Collaboration reported a charmonium-like state  $Y(4360)$  in the  $\psi(2S)\pi^+\pi^-$  invariant mass spectrum of the  $e^+e^- \rightarrow \psi(2S)\pi^+\pi^-$  process [8]. Some authors proposed  $Y(4360)$  as an isoscalar  $D_1\bar{D}^*$  molecular state [39]. Our pervious works suggested that the decay modes  $J/\psi\sigma$ ,  $J/\psi f_0(980)$ ,  $\psi(2S)\sigma$ ,  $\psi(1^3D_1)\sigma$  and  $\psi(2S)f_0(980)$  of the  $0^-(1^{--})$   $D_1\bar{D}^*$  and  $D'_1\bar{D}^*$  molecular states are suppressed [67, 68].

$Y(4360)$  may be the mixture of the pure  $D_1\bar{D}^*$  and  $D'_1\bar{D}^*$  molecular states. Considering this mixing effect, the decay modes  $J/\psi\sigma$ ,  $J/\psi f_0(980)$ ,  $\psi(2S)\sigma$ ,  $\psi(1^3D_1)\sigma$  and  $\psi(2S)f_0(980)$  are also suppressed in heavy quark symmetry limit. Thus, the molecule assumption of  $Y(4360)$  is disfavored.

The decay modes  $J/\psi\sigma$ ,  $\psi(1^3D_1)\sigma$ ,  $\psi(2S)\sigma$ ,  $J/\psi f_0(980)$  and  $\psi(2S)f_0(980)$  of the  $1^{--}$  isoscalar tetraquarks of type II are strongly suppressed. Considering its discovery mode  $\psi(2S)\pi^+\pi^-$ ,  $Y(4360)$  may be the  $1^{--}$  isoscalar tetraquarks of type I.

The decay mode  $h_c\eta$  is allowed for the  $0^-(1^{--})$   $\{^1P_1\}$  of type I, while it is not allowed for the  $0^-(1^{--})$   $\{^3P_1\}$  and  $0^-(1^{--})$   $\{^5P_1\}$  of type I. This decay mode can be employed to judge whether  $Y(4360)$  is the tetraquark state of  $0^-(1^{--})$   $\{^1P_1\}$ ,  $0^-(1^{--})$   $\{^3P_1\}$  or  $0^-(1^{--})$   $\{^5P_1\}$ .

### D. $Z_c(4200)$

Recently, the charged charmonium-like state  $Z_c(4200)^+$  was reported by Belle Collaboration in the  $Z_c(4200)^+ \rightarrow J/\psi\pi^+$  process. Its  $J^P$  quantum numbers are probably  $1^+$ . The neutral part of  $Z_c(4200)$  have quantum numbers  $I^G(J^{PC}) = 1^+(1^{+-})$ . Its mass and decay width are  $M = 4196^{+31+17}_{-29-13}$  MeV and  $\Gamma = 370^{+70+70}_{-70-132}$  MeV. The decay width of  $Z_c(4200)$  is too large to be interpreted as a molecule. Moreover, its mass is not close to the open charm threshold.

It was proposed as a S-wave tetraquark [74]. From Table III, the decay modes  $J/\psi\pi$  and  $\eta_c\rho$  are suppressed for the  $1^+(1^{+-})$  tetraquarks with the  $280_{cs}$  color-spin representation. Therefore,  $Z_c(4200)^+$  is a good candidate of the  $1^+(1^{+-})$  tetraquarks with the  $35_{cs}$  color-spin representation. If so, its P-wave decay mode  $h_c\pi$  is strongly suppressed in the heavy quark limit, while the P-wave decay modes  $\chi_{c0}\rho$  and  $\chi_{c1}\rho$  are allowed.

## VII. SUMMARY

More and more charmonium-like XYZ states have been reported in the recent years. Some of these exotic state are hard to be accommodated into the charmonium spectrum in the conventional quark model. Some of them are even charged. Various theoretical interpretations were proposed such as the  $c\bar{c}$  states distorted by couple-channel effects, hybrid charmonium, di-meson molecules, tetraquarks, and non-resonant kinematics artifacts caused by threshold effects. The tetraquark scheme is attractive for those XYZ states which do not sit on the di-meson threshold and are very broad with a width around several hundred MeV.

The charm quark mass is much larger than  $\Lambda_{QCD}$  and up and down quark mass. The heavy quark symmetry is a very powerful tool to study the charmonium-like states. In heavy quark symmetry limit, the heavy quark spin is conserved during the decay process. The color-spin rearrangement scheme is based on the conservations of heavy spin, light spin, parity, C parity and G parity, which provides us an effective framework to probe the inner structures of these exotic states from their decay behaviors.

In this work we have performed an extensive investigations of the decay patterns of the S-wave tetraquark and P-wave tetraquark states with the color-spin rearrangement scheme. We considered two kinds of P-wave tetraquarks where the P-wave excitation exists between the diquark and anti-diquark pair or inside one of the diquark. We notice the second type of P-wave tetraquark do not decay into  $J/\psi\pi^+\pi^-$  while the first type decays into this mode easily. The  $J/\psi\pi^+\pi^-$  mode is also strongly suppressed if  $Y(4260)$  is a molecular state. The decay patterns of the hybrid charmonium and hidden-charm tetraquark states very similar. Both these structures allow the open-charm decay modes, which have not been observed experimentally. This feature is very puzzling. We also discussed the decay patterns of  $X(3872)$ ,  $Y(4360)$ , and several charged states such as  $Z_c(3900)$ . The observation of in the  $h_c\pi$  mode disfavors the tetraquark assumption of  $Z_c(4020)$ .

The strong decay behaviors of the charmonium-like XYZ states encode important information on their inner structures. The experiment measurement of their decay patterns will shed light on their underlying dynamics. Hopefully the present study on the tetraquark schemes will help us illuminate the puzzling situation of the XYZ states.



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### Appendix A: Color-spin wave functions of the S-wave tetraquarks

$0^{++}$

$$|1_{cs}, 1_c, 0\rangle_{(21, \bar{2}1)} = \frac{\sqrt{21}}{6}(1, 1, 0)_H^{++} \otimes (1, 1, 0)_l^{--} - \frac{\sqrt{7}}{14}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{--} \\ + \frac{\sqrt{42}}{21}(35, 8, 0)_H^{++} \otimes (35, 8, 0)_l^{--} - \frac{\sqrt{14}}{7}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{--}, \quad (32)$$

$$|405_{cs}, 1_c, 0\rangle_{(21, \bar{2}1)} = -\frac{2\sqrt{42}}{21}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{--} + \frac{3\sqrt{7}}{14}(35, 8, 0)_H^{++} \otimes (35, 8, 0)_l^{--} \\ + \frac{5\sqrt{21}}{42}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{--}, \quad (33)$$

$$|1_{cs}, 1_c, 0\rangle_{(15, \bar{1}5)} = \frac{\sqrt{15}}{6}(1, 1, 0)_H^{++} \otimes (1, 1, 0)_l^{--} - \frac{\sqrt{5}}{10}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{--} \\ - \frac{\sqrt{30}}{15}(35, 8, 0)_H^{++} \otimes (35, 8, 0)_l^{--} - \frac{\sqrt{10}}{5}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{--}, \quad (34)$$

$$|189_{cs}, 1_c, 0\rangle_{(15, \bar{1}5)} = -\frac{2\sqrt{30}}{15}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{--} - \frac{3\sqrt{5}}{10}(35, 8, 0)_H^{++} \otimes (35, 8, 0)_l^{--} \\ - \frac{\sqrt{15}}{30}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{--}, \quad (35)$$

$1^{++}$

$$|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)} = -\frac{2\sqrt{2}}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{--} - \frac{1}{3}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{--} \quad (36)$$

$$|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)} = -\frac{1}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{--} + \frac{2\sqrt{2}}{3}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{--} \quad (37)$$

$$|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)} = -\frac{2\sqrt{2}}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{--} - \frac{1}{3}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{--} \quad (38)$$

$$|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)} = -\frac{1}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{--} + \frac{2\sqrt{2}}{3}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{--}, \quad (39)$$

$1^{+-}$

$$|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)} = \frac{\sqrt{2}}{2}(35, 1, 1)_{H^-}^- \otimes (1, 1, 0)_l^{+-} - \frac{\sqrt{2}}{2}(1, 1, 0)_{H^+}^- \otimes (35, 1, 1)_l^{--} \quad (40)$$

$$|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)} = \frac{\sqrt{2}}{2}(35, 1, 1)_{H^-}^- \otimes (1, 1, 0)_l^{+-} - \frac{\sqrt{2}}{2}(1, 1, 0)_{H^+}^- \otimes (35, 1, 1)_l^{--} \quad (41)$$

$$|35_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)} = \frac{\sqrt{3}}{3}(35, 1, 1)_{H^-}^- \otimes (1, 1, 0)_l^{+-} + \frac{\sqrt{6}}{6}(35, 8, 1)_{H^-}^- \otimes (35, 8, 0)_l^{+-} \\ + \frac{\sqrt{3}}{3}(1, 1, 0)_{H^+}^- \otimes (35, 1, 1)_l^{--} + \frac{\sqrt{6}}{6}(35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{--} \quad (42)$$

$$|35_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)} = \frac{\sqrt{6}}{6}(35, 1, 1)_{H^-}^- \otimes (1, 1, 0)_l^{+-} - \frac{\sqrt{3}}{3}(35, 8, 1)_{H^-}^- \otimes (35, 8, 0)_l^{+-} \\ + \frac{\sqrt{6}}{6}(1, 1, 0)_{H^+}^- \otimes (35, 1, 1)_l^{--} - \frac{\sqrt{3}}{3}(35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{--} \quad (43)$$

$$|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)} = -\frac{\sqrt{2}}{2}(35, 8, 1)_{H^-}^- \otimes (35, 8, 0)_l^{+-} + \frac{\sqrt{2}}{2}(35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{--} \quad (44)$$

$$|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)} = -\frac{\sqrt{2}}{2}(35, 8, 1)_{H^-}^- \otimes (35, 8, 0)_l^{+-} + \frac{\sqrt{2}}{2}(35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{--}, \quad (45)$$

### Appendix B: Color-spin wave functions of the P-wave tetraquarks

$1^{--}({}^3P_1)$  of type II

$$|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)} = \frac{\sqrt{2}}{2}(35, 1, 1)_{H^-}^- \otimes (1, 1, 1)_l^{++} - \frac{\sqrt{2}}{2}(1, 1, 0)_{H^+}^- \otimes (35, 1, 1)_l^{+-} \quad (46)$$

$$|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)} = \frac{\sqrt{2}}{2}(35, 1, 1)_{H^-}^- \otimes (1, 1, 1)_l^{++} - \frac{\sqrt{2}}{2}(1, 1, 0)_{H^+}^- \otimes (35, 1, 1)_l^{+-} \quad (47)$$

$$|35_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)} = \frac{\sqrt{3}}{3}(35, 1, 1)_{H^-}^- \otimes (1, 1, 1)_l^{++} + \frac{\sqrt{6}}{6}(35, 8, 1)_{H^-}^- \otimes (35, 8, 1)_l^{++} \\ + \frac{\sqrt{3}}{3}(1, 1, 0)_{H^+}^- \otimes (35, 1, 1)_l^{+-} + \frac{\sqrt{6}}{6}(35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{+-} \quad (48)$$

$$|35_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)} = \frac{\sqrt{6}}{6}(35, 1, 1)_{H^-}^- \otimes (1, 1, 1)_l^{++} - \frac{\sqrt{3}}{3}(35, 8, 1)_{H^-}^- \otimes (35, 8, 1)_l^{++} \\ + \frac{\sqrt{6}}{6}(1, 1, 0)_{H^+}^- \otimes (35, 1, 1)_l^{+-} - \frac{\sqrt{3}}{3}(35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{+-} \quad (49)$$

$$|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)} = -\frac{\sqrt{2}}{2}(35, 8, 1)_{H^-}^- \otimes (35, 8, 1)_l^{++} + \frac{\sqrt{2}}{2}(35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{+-} \quad (50)$$

$$|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)} = -\frac{\sqrt{2}}{2}(35, 8, 1)_{H^-}^- \otimes (35, 8, 1)_l^{++} + \frac{\sqrt{2}}{2}(35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{+-}, \quad (51)$$

$1^{--}({}^1P_1)$  of type I

$$\begin{aligned}
|1_{cs}, 1_c, 0\rangle_{(21, \bar{2}1)} &= \frac{\sqrt{21}}{6} (1, 1, 0)_{H^+}^- \otimes (1, 1, 1)_l^{+-} - \frac{\sqrt{7}}{42} (35, 1, 1)_{H^-}^- \otimes (35, 1, 0)_l^{++} \\
&+ \frac{\sqrt{21}}{42} (35, 1, 1)_{H^-}^- \otimes (35, 1, 1)_l^{++} - \frac{\sqrt{35}}{42} (35, 1, 1)_{H^-}^- \otimes (35, 1, 2)_l^{++} \\
&+ \frac{\sqrt{42}}{21} (35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{+-} - \frac{\sqrt{7}}{42} (35, 8, 1)_{H^-}^- \otimes (35, 8, 0)_l^{++} \\
&+ \frac{\sqrt{21}}{42} (35, 8, 1)_{H^-}^- \otimes (35, 8, 1)_l^{++} - \frac{\sqrt{35}}{42} (35, 8, 1)_{H^-}^- \otimes (35, 8, 2)_l^{++}
\end{aligned} \tag{52}$$

$$\begin{aligned}
|405_{cs}, 1_c, 0\rangle_{(21, \bar{2}1)} &= -\frac{2\sqrt{42}}{63} (35, 1, 1)_{H^-}^- \otimes (35, 1, 0)_l^{++} + \frac{2\sqrt{14}}{21} (35, 1, 1)_{H^-}^- \otimes (35, 1, 1)_l^{++} \\
&- \frac{2\sqrt{210}}{63} (35, 1, 1)_{H^-}^- \otimes (35, 1, 2)_l^{++} + \frac{3\sqrt{7}}{14} (35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{+-} \\
&- \frac{5\sqrt{21}}{126} (35, 8, 1)_{H^-}^- \otimes (35, 8, 0)_l^{++} - \frac{5\sqrt{7}}{42} (35, 8, 1)_{H^-}^- \otimes (35, 8, 1)_l^{++} \\
&+ \frac{5\sqrt{105}}{126} (35, 8, 1)_{H^-}^- \otimes (35, 8, 2)_l^{++}
\end{aligned} \tag{53}$$

$$\begin{aligned}
|1_{cs}, 1_c, 0\rangle_{(15, \bar{1}5)} &= \frac{\sqrt{15}}{6} (1, 1, 0)_{H^+}^- \otimes (1, 1, 1)_l^{+-} - \frac{\sqrt{5}}{30} (35, 1, 1)_{H^-}^- \otimes (35, 1, 0)_l^{++} \\
&- \frac{\sqrt{15}}{30} (35, 1, 1)_{H^-}^- \otimes (35, 1, 1)_l^{++} + \frac{1}{6} (35, 1, 1)_{H^-}^- \otimes (35, 1, 2)_l^{++} \\
&- \frac{\sqrt{30}}{15} (35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{+-} + \frac{\sqrt{10}}{15} (35, 8, 1)_{H^-}^- \otimes (35, 8, 0)_l^{++} \\
&- \frac{\sqrt{30}}{15} (35, 8, 1)_{H^-}^- \otimes (35, 8, 1)_l^{++} + \frac{\sqrt{2}}{3} (35, 8, 1)_{H^-}^- \otimes (35, 8, 2)_l^{++}
\end{aligned} \tag{54}$$

$$\begin{aligned}
|189_{cs}, 1_c, 0\rangle_{(15, \bar{1}5)} &= -\frac{2\sqrt{30}}{45} (35, 1, 1)_{H^-}^- \otimes (35, 1, 0)_l^{++} + \frac{2\sqrt{10}}{15} (35, 1, 1)_{H^-}^- \otimes (35, 1, 1)_l^{++} \\
&- \frac{2\sqrt{6}}{9} (35, 1, 1)_{H^-}^- \otimes (35, 1, 2)_l^{++} - \frac{3\sqrt{5}}{10} (35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{+-} \\
&- \frac{\sqrt{15}}{90} (35, 8, 1)_{H^-}^- \otimes (35, 8, 0)_l^{++} + \frac{\sqrt{5}}{30} (35, 8, 1)_{H^-}^- \otimes (35, 8, 1)_l^{++} \\
&- \frac{\sqrt{3}}{18} (35, 8, 1)_{H^-}^- \otimes (35, 8, 2)_l^{++},
\end{aligned} \tag{55}$$

$1^{--}({}^3P_1)$  of type I

$$\begin{aligned}
|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)} &= \frac{2\sqrt{6}}{9}(35, 8, 1)_{H}^{--} \otimes (35, 8, 0)_l^{++} - \frac{\sqrt{2}}{3}(35, 8, 1)_{H}^{--} \otimes (35, 8, 1)_l^{++} \\
&\quad - \frac{\sqrt{30}}{9}(35, 8, 1)_{H}^{--} \otimes (35, 8, 2)_l^{++} \\
&\quad + \frac{\sqrt{3}}{9}(35, 1, 1)_{H}^{--} \otimes (35, 1, 0)_l^{++} - \frac{1}{6}(35, 1, 1)_{H}^{--} \otimes (35, 1, 1)_l^{++} \\
&\quad - \frac{\sqrt{15}}{18}(35, 1, 1)_{H}^{--} \otimes (35, 1, 2)_l^{++}
\end{aligned} \tag{56}$$

$$\begin{aligned}
|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)} &= \frac{2\sqrt{6}}{9}(35, 8, 1)_{H}^{--} \otimes (35, 8, 0)_l^{++} - \frac{\sqrt{2}}{3}(35, 8, 1)_{H}^{--} \otimes (35, 8, 1)_l^{++} \\
&\quad - \frac{\sqrt{30}}{9}(35, 8, 1)_{H}^{--} \otimes (35, 8, 2)_l^{++} \\
&\quad + \frac{\sqrt{3}}{9}(35, 1, 1)_{H}^{--} \otimes (35, 1, 0)_l^{++} - \frac{1}{6}(35, 1, 1)_{H}^{--} \otimes (35, 1, 1)_l^{++} \\
&\quad - \frac{\sqrt{15}}{18}(35, 1, 1)_{H}^{--} \otimes (35, 1, 2)_l^{++}
\end{aligned} \tag{57}$$

$$\begin{aligned}
|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)} &= \frac{\sqrt{3}}{9}(35, 8, 1)_{H}^{--} \otimes (35, 8, 0)_l^{++} - \frac{1}{6}(35, 8, 1)_{H}^{--} \otimes (35, 8, 1)_l^{++} \\
&\quad - \frac{\sqrt{15}}{18}(35, 8, 1)_{H}^{--} \otimes (35, 8, 2)_l^{++} \\
&\quad - \frac{2\sqrt{6}}{9}(35, 1, 1)_{H}^{--} \otimes (35, 1, 0)_l^{++} + \frac{\sqrt{2}}{3}(35, 1, 1)_{H}^{--} \otimes (35, 1, 1)_l^{++} \\
&\quad + \frac{\sqrt{30}}{9}(35, 1, 1)_{H}^{--} \otimes (35, 1, 2)_l^{++}
\end{aligned} \tag{58}$$

$$\begin{aligned}
|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)} &= \frac{\sqrt{3}}{9}(35, 8, 1)_{H}^{--} \otimes (35, 8, 0)_l^{++} - \frac{1}{6}(35, 8, 1)_{H}^{--} \otimes (35, 8, 1)_l^{++} \\
&\quad - \frac{\sqrt{15}}{18}(35, 8, 1)_{H}^{--} \otimes (35, 8, 2)_l^{++} \\
&\quad - \frac{2\sqrt{6}}{9}(35, 1, 1)_{H}^{--} \otimes (35, 1, 0)_l^{++} + \frac{\sqrt{2}}{3}(35, 1, 1)_{H}^{--} \otimes (35, 1, 1)_l^{++} \\
&\quad + \frac{\sqrt{30}}{9}(35, 1, 1)_{H}^{--} \otimes (35, 1, 2)_l^{++},
\end{aligned} \tag{59}$$

$1^{--}({}^5P_1)$  of type I



$$\begin{aligned}
|405_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)} &= \frac{\sqrt{15}}{9} (35, 8, 1)_H^{--} \otimes (35, 8, 0)_l^{++} + \frac{\sqrt{5}}{6} (35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{++} \\
&+ \frac{\sqrt{3}}{18} (35, 8, 1)_H^{--} \otimes (35, 8, 2)_l^{++} \\
&+ \frac{\sqrt{30}}{9} (35, 1, 1)_H^{--} \otimes (35, 1, 0)_l^{++} - \frac{\sqrt{10}}{6} (35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{++} \\
&+ \frac{\sqrt{6}}{18} (35, 1, 1)_H^{--} \otimes (35, 1, 2)_l^{++}
\end{aligned} \tag{60}$$

$$\begin{aligned}
|189_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)} &= \frac{\sqrt{30}}{9} (35, 8, 1)_H^{--} \otimes (35, 8, 0)_l^{++} + \frac{\sqrt{10}}{6} (35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{++} \\
&+ \frac{\sqrt{6}}{18} (35, 8, 1)_H^{--} \otimes (35, 8, 2)_l^{++} \\
&- \frac{\sqrt{15}}{9} (35, 1, 1)_H^{--} \otimes (35, 1, 0)_l^{++} - \frac{\sqrt{5}}{6} (35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{++} \\
&- \frac{\sqrt{3}}{18} (35, 1, 1)_H^{--} \otimes (35, 1, 2)_l^{++},
\end{aligned} \tag{61}$$

$1^{--}$  of type I

$$|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)} = \frac{\sqrt{2}}{2} (35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{+-} - \frac{\sqrt{2}}{2} (1, 1, 0)_H^{+-} \otimes (35, 1, 1)_l^{++} \tag{62}$$

$$|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)} = \frac{\sqrt{2}}{2} (35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{+-} - \frac{\sqrt{2}}{2} (1, 1, 0)_H^{+-} \otimes (35, 1, 1)_l^{++} \tag{63}$$

$$\begin{aligned}
|35_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)} &= \frac{\sqrt{3}}{3} (35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{+-} + \frac{\sqrt{6}}{6} (35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} \\
&+ \frac{\sqrt{3}}{3} (1, 1, 0)_H^{+-} \otimes (35, 1, 1)_l^{++} + \frac{\sqrt{6}}{6} (35, 8, 0)_H^{+-} \otimes (35, 8, 1)_l^{++}
\end{aligned} \tag{64}$$

$$\begin{aligned}
|35_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)} &= \frac{\sqrt{6}}{6} (35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{+-} - \frac{\sqrt{3}}{3} (35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} \\
&+ \frac{\sqrt{6}}{6} (1, 1, 0)_H^{+-} \otimes (35, 1, 1)_l^{++} - \frac{\sqrt{3}}{3} (35, 8, 0)_H^{+-} \otimes (35, 8, 1)_l^{++}
\end{aligned} \tag{65}$$

$$|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)} = -\frac{\sqrt{2}}{2} (35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} + \frac{\sqrt{2}}{2} (35, 8, 0)_H^{+-} \otimes (35, 8, 1)_l^{++} \tag{66}$$

$$|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)} = -\frac{\sqrt{2}}{2} (35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} + \frac{\sqrt{2}}{2} (35, 8, 0)_H^{+-} \otimes (35, 8, 1)_l^{++}, \tag{67}$$

$1^{--}({}^1P_1)$  of type II

$$\begin{aligned}
|1_{cs}, 1_c, 0\rangle_{(21, \bar{2}1)} &= \frac{\sqrt{21}}{6} (1, 1, 0)_{H^+}^- \otimes (1, 1, 1)_l^{++} - \frac{\sqrt{7}}{42} (35, 1, 1)_{H^-}^- \otimes (35, 1, 0)_l^{+-} \\
&+ \frac{\sqrt{21}}{42} (35, 1, 1)_{H^-}^- \otimes (35, 1, 1)_l^{+-} - \frac{\sqrt{35}}{42} (35, 1, 1)_{H^-}^- \otimes (35, 1, 2)_l^{+-} \\
&+ \frac{\sqrt{42}}{21} (35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{++} - \frac{\sqrt{7}}{42} (35, 8, 1)_{H^-}^- \otimes (35, 8, 0)_l^{+-} \\
&+ \frac{\sqrt{21}}{42} (35, 8, 1)_{H^-}^- \otimes (35, 8, 1)_l^{+-} - \frac{\sqrt{35}}{42} (35, 8, 1)_{H^-}^- \otimes (35, 8, 2)_l^{+-}
\end{aligned} \tag{68}$$

$$\begin{aligned}
|405_{cs}, 1_c, 0\rangle_{(21, \bar{2}1)} &= -\frac{2\sqrt{42}}{63} (35, 1, 1)_{H^-}^- \otimes (35, 1, 0)_l^{+-} + \frac{2\sqrt{14}}{21} (35, 1, 1)_{H^-}^- \otimes (35, 1, 1)_l^{+-} \\
&- \frac{2\sqrt{210}}{63} (35, 1, 1)_{H^-}^- \otimes (35, 1, 2)_l^{+-} + \frac{3\sqrt{7}}{14} (35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{++} \\
&- \frac{5\sqrt{21}}{126} (35, 8, 1)_{H^-}^- \otimes (35, 8, 0)_l^{+-} - \frac{5\sqrt{7}}{42} (35, 8, 1)_{H^-}^- \otimes (35, 8, 1)_l^{+-} \\
&+ \frac{5\sqrt{105}}{126} (35, 8, 1)_{H^-}^- \otimes (35, 8, 2)_l^{+-}
\end{aligned} \tag{69}$$

$$\begin{aligned}
|1_{cs}, 1_c, 0\rangle_{(15, \bar{1}5)} &= \frac{\sqrt{15}}{6} (1, 1, 0)_{H^+}^- \otimes (1, 1, 1)_l^{++} - \frac{\sqrt{5}}{30} (35, 1, 1)_{H^-}^- \otimes (35, 1, 0)_l^{+-} \\
&- \frac{\sqrt{15}}{30} (35, 1, 1)_{H^-}^- \otimes (35, 1, 1)_l^{+-} + \frac{1}{6} (35, 1, 1)_{H^-}^- \otimes (35, 1, 2)_l^{+-} \\
&- \frac{\sqrt{30}}{15} (35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{++} + \frac{\sqrt{10}}{15} (35, 8, 1)_{H^-}^- \otimes (35, 8, 0)_l^{+-} \\
&- \frac{\sqrt{30}}{15} (35, 8, 1)_{H^-}^- \otimes (35, 8, 1)_l^{+-} + \frac{\sqrt{2}}{3} (35, 8, 1)_{H^-}^- \otimes (35, 8, 2)_l^{+-}
\end{aligned} \tag{70}$$

$$\begin{aligned}
|189_{cs}, 1_c, 0\rangle_{(15, \bar{1}5)} &= -\frac{2\sqrt{30}}{45} (35, 1, 1)_{H^-}^- \otimes (35, 1, 0)_l^{+-} + \frac{2\sqrt{10}}{15} (35, 1, 1)_{H^-}^- \otimes (35, 1, 1)_l^{+-} \\
&- \frac{2\sqrt{6}}{9} (35, 1, 1)_{H^-}^- \otimes (35, 1, 2)_l^{+-} - \frac{3\sqrt{5}}{10} (35, 8, 0)_{H^+}^- \otimes (35, 8, 1)_l^{++} \\
&- \frac{\sqrt{15}}{90} (35, 8, 1)_{H^-}^- \otimes (35, 8, 0)_l^{+-} + \frac{\sqrt{5}}{30} (35, 8, 1)_{H^-}^- \otimes (35, 8, 1)_l^{+-} \\
&- \frac{\sqrt{3}}{18} (35, 8, 1)_{H^-}^- \otimes (35, 8, 2)_l^{+-},
\end{aligned} \tag{71}$$

$1^{-+}({}^3P_1)$  of type II

$$\begin{aligned}
|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)} &= \frac{2\sqrt{6}}{9}(35, 8, 1)_{H}^{--} \otimes (35, 8, 0)_l^{+-} - \frac{\sqrt{2}}{3}(35, 8, 1)_{H}^{--} \otimes (35, 8, 1)_l^{+-} \\
&\quad - \frac{\sqrt{30}}{9}(35, 8, 1)_{H}^{--} \otimes (35, 8, 2)_l^{+-} \\
&\quad + \frac{\sqrt{3}}{9}(35, 1, 1)_{H}^{--} \otimes (35, 1, 0)_l^{+-} - \frac{1}{6}(35, 1, 1)_{H}^{--} \otimes (35, 1, 1)_l^{+-} \\
&\quad - \frac{\sqrt{15}}{18}(35, 1, 1)_{H}^{--} \otimes (35, 1, 2)_l^{+-} \tag{72}
\end{aligned}$$

$$\begin{aligned}
|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)} &= \frac{2\sqrt{6}}{9}(35, 8, 1)_{H}^{--} \otimes (35, 8, 0)_l^{+-} - \frac{\sqrt{2}}{3}(35, 8, 1)_{H}^{--} \otimes (35, 8, 1)_l^{+-} \\
&\quad - \frac{\sqrt{30}}{9}(35, 8, 1)_{H}^{--} \otimes (35, 8, 2)_l^{+-} \\
&\quad + \frac{\sqrt{3}}{9}(35, 1, 1)_{H}^{--} \otimes (35, 1, 0)_l^{+-} - \frac{1}{6}(35, 1, 1)_{H}^{--} \otimes (35, 1, 1)_l^{+-} \\
&\quad - \frac{\sqrt{15}}{18}(35, 1, 1)_{H}^{--} \otimes (35, 1, 2)_l^{+-} \tag{73}
\end{aligned}$$

$$\begin{aligned}
|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)} &= \frac{\sqrt{3}}{9}(35, 8, 1)_{H}^{--} \otimes (35, 8, 0)_l^{+-} - \frac{1}{6}(35, 8, 1)_{H}^{--} \otimes (35, 8, 1)_l^{+-} \\
&\quad - \frac{\sqrt{15}}{18}(35, 8, 1)_{H}^{--} \otimes (35, 8, 2)_l^{+-} \\
&\quad - \frac{2\sqrt{6}}{9}(35, 1, 1)_{H}^{--} \otimes (35, 1, 0)_l^{+-} + \frac{\sqrt{2}}{3}(35, 1, 1)_{H}^{--} \otimes (35, 1, 1)_l^{+-} \\
&\quad + \frac{\sqrt{30}}{9}(35, 1, 1)_{H}^{--} \otimes (35, 1, 2)_l^{+-} \tag{74}
\end{aligned}$$

$$\begin{aligned}
|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)} &= \frac{\sqrt{3}}{9}(35, 8, 1)_{H}^{--} \otimes (35, 8, 0)_l^{+-} - \frac{1}{6}(35, 8, 1)_{H}^{--} \otimes (35, 8, 1)_l^{+-} \\
&\quad - \frac{\sqrt{15}}{18}(35, 8, 1)_{H}^{--} \otimes (35, 8, 2)_l^{+-} \\
&\quad - \frac{2\sqrt{6}}{9}(35, 1, 1)_{H}^{--} \otimes (35, 1, 0)_l^{+-} + \frac{\sqrt{2}}{3}(35, 1, 1)_{H}^{--} \otimes (35, 1, 1)_l^{+-} \\
&\quad + \frac{\sqrt{30}}{9}(35, 1, 1)_{H}^{--} \otimes (35, 1, 2)_l^{+-}, \tag{75}
\end{aligned}$$

$1^{-+}({}^5P_1)$ of type II
------------------------------

$$\begin{aligned}
|405_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)} &= \frac{\sqrt{15}}{9}(35, 8, 1)_H^{--} \otimes (35, 8, 0)_l^{+-} + \frac{\sqrt{5}}{6}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} \\
&+ \frac{\sqrt{3}}{18}(35, 8, 1)_H^{--} \otimes (35, 8, 2)_l^{+-} \\
&+ \frac{\sqrt{30}}{9}(35, 1, 1)_H^{--} \otimes (35, 1, 0)_l^{+-} - \frac{\sqrt{10}}{6}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{+-} \\
&+ \frac{\sqrt{6}}{18}(35, 1, 1)_H^{--} \otimes (35, 1, 2)_l^{+-}
\end{aligned} \tag{76}$$

$$\begin{aligned}
|189_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)} &= \frac{\sqrt{30}}{9}(35, 8, 1)_H^{--} \otimes (35, 8, 0)_l^{+-} + \frac{\sqrt{10}}{6}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} \\
&+ \frac{\sqrt{6}}{18}(35, 8, 1)_H^{--} \otimes (35, 8, 2)_l^{+-} \\
&- \frac{\sqrt{15}}{9}(35, 1, 1)_H^{--} \otimes (35, 1, 0)_l^{+-} - \frac{\sqrt{5}}{6}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{+-} \\
&- \frac{\sqrt{3}}{18}(35, 1, 1)_H^{--} \otimes (35, 1, 2)_l^{+-},
\end{aligned} \tag{77}$$

$0^{-+}$  of type I

$$|35_{cs}, 1_c, 0\rangle_{(21, \bar{2}1)} = \frac{\sqrt{2}}{2}(35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{+-} - \frac{\sqrt{2}}{2}(1, 1, 0)_H^{++} \otimes (35, 1, 0)_l^{++} \tag{78}$$

$$|35_{cs}, 1_c, 0\rangle_{(15, \bar{1}5)} = \frac{\sqrt{2}}{2}(35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{+-} - \frac{\sqrt{2}}{2}(1, 1, 0)_H^{++} \otimes (35, 1, 0)_l^{++} \tag{79}$$

$$\begin{aligned}
|35_{cs}, 1_c, 0\rangle_{(21, \bar{1}5)} &= \frac{\sqrt{3}}{3}(35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{+-} + \frac{\sqrt{6}}{6}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} \\
&+ \frac{\sqrt{3}}{3}(1, 1, 0)_H^{++} \otimes (35, 1, 0)_l^{++} + \frac{\sqrt{6}}{6}(35, 8, 0)_H^{++} \otimes (35, 8, 0)_l^{++}
\end{aligned} \tag{80}$$

$$\begin{aligned}
|35_{cs}, 1_c, 0\rangle_{(15, \bar{2}1)} &= \frac{\sqrt{6}}{6}(35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{+-} - \frac{\sqrt{3}}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} \\
&+ \frac{\sqrt{6}}{6}(1, 1, 0)_H^{++} \otimes (35, 1, 0)_l^{++} - \frac{\sqrt{3}}{3}(35, 8, 0)_H^{++} \otimes (35, 8, 0)_l^{++}
\end{aligned} \tag{81}$$

$$|280_{cs}, 1_c, 0\rangle_{(21, \bar{1}5)} = -\frac{\sqrt{2}}{2}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} + \frac{\sqrt{2}}{2}(35, 8, 0)_H^{++} \otimes (35, 8, 0)_l^{++} \tag{82}$$

$$|280_{cs}, 1_c, 0\rangle_{(15, \bar{2}1)} = -\frac{\sqrt{2}}{2}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} + \frac{\sqrt{2}}{2}(35, 8, 0)_H^{++} \otimes (35, 8, 0)_l^{++}, \tag{83}$$

$0^{-+}$  of type II

$$|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)} = -\frac{2\sqrt{2}}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} - \frac{1}{3}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{+-} \tag{84}$$

$$|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)} = -\frac{1}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} + \frac{2\sqrt{2}}{3}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{+-} \tag{85}$$

$$|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)} = -\frac{2\sqrt{2}}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} - \frac{1}{3}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{+-} \tag{86}$$

$$|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)} = -\frac{1}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{+-} + \frac{2\sqrt{2}}{3}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{+-}, \tag{87}$$

$0^{--}$  of type I

$$|35_{cs}, 1_c, 1\rangle_{(21, \bar{2}1)} = -\frac{2\sqrt{2}}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{++} - \frac{1}{3}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{++} \quad (88)$$

$$|280_{cs}, 1_c, 1\rangle_{(21, \bar{1}5)} = -\frac{1}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{++} + \frac{2\sqrt{2}}{3}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{++} \quad (89)$$

$$|35_{cs}, 1_c, 1\rangle_{(15, \bar{1}5)} = -\frac{2\sqrt{2}}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{++} - \frac{1}{3}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{++} \quad (90)$$

$$|280_{cs}, 1_c, 1\rangle_{(15, \bar{2}1)} = -\frac{1}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{++} + \frac{2\sqrt{2}}{3}(35, 1, 1)_H^{--} \otimes (35, 1, 1)_l^{++}, \quad (91)$$

$0^{--}$  of type II

$$|35_{cs}, 1_c, 0\rangle_{(21, \bar{2}1)} = \frac{\sqrt{2}}{2}(35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{++} - \frac{\sqrt{2}}{2}(1, 1, 0)_H^{--} \otimes (35, 1, 0)_l^{++} \quad (92)$$

$$|35_{cs}, 1_c, 0\rangle_{(15, \bar{1}5)} = \frac{\sqrt{2}}{2}(35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{++} - \frac{\sqrt{2}}{2}(1, 1, 0)_H^{--} \otimes (35, 1, 0)_l^{++} \quad (93)$$

$$|35_{cs}, 1_c, 0\rangle_{(21, \bar{1}5)} = \frac{\sqrt{3}}{3}(35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{++} + \frac{\sqrt{6}}{6}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{++} \\ + \frac{\sqrt{3}}{3}(1, 1, 0)_H^{--} \otimes (35, 1, 0)_l^{++} + \frac{\sqrt{6}}{6}(35, 8, 0)_H^{--} \otimes (35, 8, 0)_l^{++} \quad (94)$$

$$|35_{cs}, 1_c, 0\rangle_{(15, \bar{2}1)} = \frac{\sqrt{6}}{6}(35, 1, 1)_H^{--} \otimes (1, 1, 1)_l^{++} - \frac{\sqrt{3}}{3}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{++} \\ + \frac{\sqrt{6}}{6}(1, 1, 0)_H^{--} \otimes (35, 1, 0)_l^{++} - \frac{\sqrt{3}}{3}(35, 8, 0)_H^{--} \otimes (35, 8, 0)_l^{++} \quad (95)$$

$$|280_{cs}, 1_c, 0\rangle_{(21, \bar{1}5)} = -\frac{\sqrt{2}}{2}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{++} + \frac{\sqrt{2}}{2}(35, 8, 0)_H^{--} \otimes (35, 8, 0)_l^{++} \quad (96)$$

$$|280_{cs}, 1_c, 0\rangle_{(15, \bar{2}1)} = -\frac{\sqrt{2}}{2}(35, 8, 1)_H^{--} \otimes (35, 8, 1)_l^{++} + \frac{\sqrt{2}}{2}(35, 8, 0)_H^{--} \otimes (35, 8, 0)_l^{++}, \quad (97)$$

#### Appendix C: The parameters $A_{(1-24)}$ and $B_{(1-24)}$ in Table IV.

$$A_1 = -\frac{\sqrt{7}}{126}H_\rho^{10} - \frac{\sqrt{7}}{42}H_\rho^{11} - \frac{5\sqrt{7}}{126}H_\rho^{12} - \frac{\sqrt{7}}{126}\tilde{H}_\rho^{10} - \frac{\sqrt{7}}{42}\tilde{H}_\rho^{11} - \frac{5\sqrt{7}}{126}\tilde{H}_\rho^{12}, \\ A_2 = \frac{\sqrt{21}}{126}H_\rho^{10} + \frac{\sqrt{21}}{84}H_\rho^{11} - \frac{5\sqrt{21}}{252}H_\rho^{12} + \frac{\sqrt{21}}{126}\tilde{H}_\rho^{10} + \frac{\sqrt{21}}{84}\tilde{H}_\rho^{11} - \frac{5\sqrt{21}}{252}\tilde{H}_\rho^{12}, \\ A_3 = -\frac{\sqrt{35}}{126}H_\rho^{10} + \frac{\sqrt{35}}{84}H_\rho^{11} - \frac{\sqrt{35}}{252}H_\rho^{12} - \frac{\sqrt{35}}{126}\tilde{H}_\rho^{10} + \frac{\sqrt{35}}{84}\tilde{H}_\rho^{11} - \frac{\sqrt{35}}{252}\tilde{H}_\rho^{12},$$

$$A_4 = -\frac{2\sqrt{42}}{189}H_\rho^{10} - \frac{2\sqrt{42}}{63}H_\rho^{11} - \frac{10\sqrt{42}}{189}H_\rho^{12} - \frac{5\sqrt{21}}{378}\tilde{H}_\rho^{10} + \frac{5\sqrt{21}}{126}\tilde{H}_\rho^{11} + \frac{25\sqrt{21}}{378}\tilde{H}_\rho^{12}, \\ A_5 = \frac{2\sqrt{14}}{63}H_\rho^{10} + \frac{\sqrt{14}}{21}H_\rho^{11} - \frac{5\sqrt{14}}{63}H_\rho^{12} + \frac{5\sqrt{7}}{126}\tilde{H}_\rho^{10} - \frac{5\sqrt{7}}{84}\tilde{H}_\rho^{11} + \frac{25\sqrt{7}}{252}\tilde{H}_\rho^{12}, \\ A_6 = -\frac{2\sqrt{210}}{189}H_\rho^{10} + \frac{\sqrt{210}}{63}H_\rho^{11} - \frac{\sqrt{210}}{189}H_\rho^{12} - \frac{5\sqrt{105}}{378}\tilde{H}_\rho^{10} - \frac{5\sqrt{105}}{252}\tilde{H}_\rho^{11} + \frac{5\sqrt{105}}{756}\tilde{H}_\rho^{12},$$

$$\begin{aligned}
A_7 &= -\frac{\sqrt{5}}{90}H_\rho^{10} + \frac{\sqrt{5}}{30}H_\rho^{11} + \frac{\sqrt{5}}{18}H_\rho^{12} + \frac{\sqrt{10}}{45}\tilde{H}_\rho^{10} + \frac{\sqrt{10}}{15}\tilde{H}_\rho^{11} + \frac{\sqrt{10}}{9}\tilde{H}_\rho^{12}, \\
A_8 &= \frac{\sqrt{15}}{90}H_\rho^{10} - \frac{\sqrt{15}}{60}H_\rho^{11} + \frac{\sqrt{15}}{36}H_\rho^{12} - \frac{\sqrt{30}}{45}\tilde{H}_\rho^{10} - \frac{\sqrt{30}}{30}\tilde{H}_\rho^{11} + \frac{\sqrt{30}}{18}\tilde{H}_\rho^{12}, \\
A_9 &= -\frac{1}{18}H_\rho^{10} - \frac{1}{12}H_\rho^{11} + \frac{1}{36}H_\rho^{12} + \frac{\sqrt{2}}{9}\tilde{H}_\rho^{10} - \frac{\sqrt{2}}{6}\tilde{H}_\rho^{11} + \frac{\sqrt{2}}{18}\tilde{H}_\rho^{12}, \\
A_{10} &= -\frac{2\sqrt{30}}{135}H_\rho^{10} - \frac{2\sqrt{30}}{45}H_\rho^{11} - \frac{2\sqrt{30}}{27}H_\rho^{12} - \frac{\sqrt{15}}{270}\tilde{H}_\rho^{10} - \frac{\sqrt{15}}{90}\tilde{H}_\rho^{11} - \frac{\sqrt{15}}{54}\tilde{H}_\rho^{12}, \\
A_{11} &= \frac{2\sqrt{10}}{45}H_\rho^{10} + \frac{\sqrt{10}}{15}H_\rho^{11} - \frac{\sqrt{10}}{9}H_\rho^{12} + \frac{\sqrt{5}}{90}\tilde{H}_\rho^{10} + \frac{\sqrt{5}}{10}\tilde{H}_\rho^{11} + \frac{\sqrt{5}}{36}\tilde{H}_\rho^{12}, \\
A_{12} &= -\frac{2\sqrt{6}}{27}H_\rho^{10} + \frac{\sqrt{6}}{9}H_\rho^{11} - \frac{\sqrt{6}}{27}H_\rho^{12} - \frac{\sqrt{3}}{54}\tilde{H}_\rho^{10} + \frac{\sqrt{3}}{36}\tilde{H}_\rho^{11} - \frac{\sqrt{3}}{108}\tilde{H}_\rho^{12}, \\
A_{13} &= \frac{\sqrt{3}}{27}H_\rho^{10} + \frac{\sqrt{3}}{18}H_\rho^{11} - \frac{5\sqrt{3}}{54}H_\rho^{12} + \frac{2\sqrt{6}}{27}\tilde{H}_\rho^{10} + \frac{\sqrt{6}}{9}\tilde{H}_\rho^{11} - \frac{5\sqrt{6}}{27}\tilde{H}_\rho^{12}, \\
A_{14} &= -\frac{1}{9}H_\rho^{10} - \frac{1}{12}H_\rho^{11} - \frac{5}{36}H_\rho^{12} - \frac{2\sqrt{2}}{9}\tilde{H}_\rho^{10} - \frac{\sqrt{2}}{6}\tilde{H}_\rho^{11} - \frac{5\sqrt{2}}{18}\tilde{H}_\rho^{12}, \\
A_{15} &= \frac{\sqrt{15}}{27}H_\rho^{10} - \frac{\sqrt{15}}{36}H_\rho^{11} - \frac{\sqrt{15}}{108}H_\rho^{12} + \frac{2\sqrt{30}}{27}\tilde{H}_\rho^{10} - \frac{\sqrt{30}}{18}\tilde{H}_\rho^{11} - \frac{\sqrt{30}}{54}\tilde{H}_\rho^{12}, \\
A_{16} &= -\frac{2\sqrt{6}}{27}H_\rho^{10} - \frac{\sqrt{6}}{9}H_\rho^{11} + \frac{5\sqrt{6}}{27}H_\rho^{12} + \frac{\sqrt{3}}{27}\tilde{H}_\rho^{10} + \frac{\sqrt{3}}{18}\tilde{H}_\rho^{11} + \frac{5\sqrt{3}}{54}\tilde{H}_\rho^{12}, \\
A_{17} &= \frac{2\sqrt{2}}{9}H_\rho^{10} + \frac{\sqrt{2}}{6}H_\rho^{11} + \frac{5\sqrt{2}}{18}H_\rho^{12} - \frac{1}{9}\tilde{H}_\rho^{10} - \frac{1}{12}\tilde{H}_\rho^{11} - \frac{5}{36}\tilde{H}_\rho^{12}, \\
A_{18} &= -\frac{2\sqrt{30}}{27}H_\rho^{10} + \frac{\sqrt{30}}{18}H_\rho^{11} + \frac{\sqrt{30}}{54}H_\rho^{12} + \frac{\sqrt{15}}{27}\tilde{H}_\rho^{10} - \frac{\sqrt{15}}{36}\tilde{H}_\rho^{11} - \frac{\sqrt{15}}{108}\tilde{H}_\rho^{12}, \\
A_{19} &= \frac{\sqrt{30}}{27}H_\rho^{10} + \frac{\sqrt{30}}{18}H_\rho^{11} + \frac{\sqrt{30}}{54}H_\rho^{12} + \frac{\sqrt{15}}{27}\tilde{H}_\rho^{10} - \frac{\sqrt{15}}{18}\tilde{H}_\rho^{11} + \frac{\sqrt{15}}{54}\tilde{H}_\rho^{12}, \\
A_{20} &= -\frac{\sqrt{10}}{9}H_\rho^{10} - \frac{\sqrt{10}}{12}H_\rho^{11} + \frac{\sqrt{10}}{36}H_\rho^{12} - \frac{\sqrt{5}}{9}\tilde{H}_\rho^{10} + \frac{\sqrt{5}}{12}\tilde{H}_\rho^{11} + \frac{\sqrt{5}}{36}\tilde{H}_\rho^{12}, \\
A_{21} &= \frac{5\sqrt{6}}{27}H_\rho^{10} + \frac{5\sqrt{6}}{36}H_\rho^{11} + \frac{\sqrt{6}}{108}H_\rho^{12} + \frac{5\sqrt{3}}{27}\tilde{H}_\rho^{10} + \frac{5\sqrt{3}}{36}\tilde{H}_\rho^{11} + \frac{\sqrt{3}}{108}\tilde{H}_\rho^{12}, \\
A_{22} &= -\frac{\sqrt{15}}{27}H_\rho^{10} + \frac{\sqrt{15}}{18}H_\rho^{11} - \frac{\sqrt{15}}{54}H_\rho^{12} + \frac{\sqrt{30}}{27}\tilde{H}_\rho^{10} - \frac{\sqrt{30}}{18}\tilde{H}_\rho^{11} + \frac{\sqrt{30}}{54}\tilde{H}_\rho^{12}, \\
A_{23} &= \frac{\sqrt{5}}{9}H_\rho^{10} + \frac{\sqrt{5}}{12}H_\rho^{11} - \frac{\sqrt{5}}{36}H_\rho^{12} - \frac{\sqrt{10}}{9}\tilde{H}_\rho^{10} + \frac{\sqrt{10}}{12}\tilde{H}_\rho^{11} + \frac{\sqrt{10}}{36}\tilde{H}_\rho^{12}, \\
A_{24} &= -\frac{5\sqrt{3}}{27}H_\rho^{10} - \frac{5\sqrt{3}}{36}H_\rho^{11} - \frac{\sqrt{3}}{108}H_\rho^{12} + \frac{5\sqrt{6}}{27}\tilde{H}_\rho^{10} + \frac{5\sqrt{6}}{36}\tilde{H}_\rho^{11} + \frac{\sqrt{6}}{108}\tilde{H}_\rho^{12}, \\
A_{25} &= -\frac{\sqrt{15}}{27}H_\rho^{10} + \frac{\sqrt{15}}{18}H_\rho^{11} - \frac{\sqrt{15}}{54}H_\rho^{12} + \frac{\sqrt{30}}{27}\tilde{H}_\rho^{10} - \frac{\sqrt{30}}{18}\tilde{H}_\rho^{11} + \frac{\sqrt{30}}{54}\tilde{H}_\rho^{12}, \\
A_{26} &= \frac{\sqrt{5}}{9}H_\rho^{10} + \frac{\sqrt{5}}{12}H_\rho^{11} - \frac{\sqrt{5}}{36}H_\rho^{12} - \frac{\sqrt{10}}{9}\tilde{H}_\rho^{10} + \frac{\sqrt{10}}{12}\tilde{H}_\rho^{11} + \frac{\sqrt{10}}{36}\tilde{H}_\rho^{12}, \\
A_{27} &= -\frac{5\sqrt{3}}{27}H_\rho^{10} - \frac{5\sqrt{3}}{36}H_\rho^{11} - \frac{\sqrt{3}}{108}H_\rho^{12} + \frac{5\sqrt{6}}{27}\tilde{H}_\rho^{10} + \frac{5\sqrt{6}}{36}\tilde{H}_\rho^{11} + \frac{\sqrt{6}}{108}\tilde{H}_\rho^{12},
\end{aligned}$$



$$\begin{aligned}
B_1 &= -\frac{\sqrt{21}}{84}H_\pi^{21} - \frac{\sqrt{105}}{84}H_\pi^{21} - \frac{\sqrt{21}}{84}\tilde{H}_\pi^{21} - \frac{\sqrt{105}}{84}\tilde{H}_\pi^{21}, \\
B_2 &= \frac{\sqrt{63}}{84}H_\pi^{21} - \frac{\sqrt{35}}{84}H_\pi^{21} + \frac{\sqrt{63}}{84}\tilde{H}_\pi^{21} - \frac{\sqrt{35}}{84}\tilde{H}_\pi^{21}, \\
B_3 &= -\frac{\sqrt{14}}{21}H_\pi^{21} - \frac{\sqrt{70}}{21}H_\pi^{21} + \frac{5\sqrt{7}}{84}\tilde{H}_\pi^{21} + \frac{5\sqrt{35}}{84}\tilde{H}_\pi^{21}, \\
B_4 &= \frac{\sqrt{42}}{21}H_\pi^{21} - \frac{\sqrt{210}}{63}H_\pi^{21} - \frac{5\sqrt{21}}{84}\tilde{H}_\pi^{21} + \frac{5\sqrt{105}}{252}\tilde{H}_\pi^{21}, \\
B_5 &= \frac{\sqrt{15}}{60}H_\pi^{21} + \frac{\sqrt{3}}{12}H_\pi^{21} + \frac{\sqrt{30}}{30}\tilde{H}_\pi^{21} + \frac{\sqrt{6}}{6}\tilde{H}_\pi^{21}, \\
B_6 &= -\frac{\sqrt{15}}{20}H_\pi^{21} + \frac{1}{12}H_\pi^{21} - \frac{\sqrt{10}}{10}\tilde{H}_\pi^{21} + \frac{\sqrt{2}}{6}\tilde{H}_\pi^{21},
\end{aligned}$$

$$\begin{aligned}
B_7 &= -\frac{\sqrt{10}}{15}H_\pi^{21} - \frac{\sqrt{2}}{3}H_\pi^{21} - \frac{\sqrt{5}}{60}\tilde{H}_\pi^{21} - \frac{1}{12}\tilde{H}_\pi^{21}, \\
B_8 &= \frac{\sqrt{30}}{15}H_\pi^{21} - \frac{\sqrt{6}}{9}H_\pi^{21} + \frac{\sqrt{15}}{60}\tilde{H}_\pi^{21} - \frac{\sqrt{3}}{36}\tilde{H}_\pi^{21}, \\
B_9 &= \frac{1}{12}H_\pi^{21} - \frac{\sqrt{5}}{12}H_\pi^{21} + \frac{\sqrt{2}}{6}\tilde{H}_\pi^{21} - \frac{\sqrt{10}}{6}\tilde{H}_\pi^{21}, \\
B_{10} &= -\frac{\sqrt{3}}{12}H_\pi^{21} - \frac{\sqrt{15}}{36}H_\pi^{21} - \frac{\sqrt{6}}{6}\tilde{H}_\pi^{21} - \frac{\sqrt{30}}{18}\tilde{H}_\pi^{21}, \\
B_{11} &= -\frac{\sqrt{2}}{6}H_\pi^{21} + \frac{\sqrt{10}}{6}H_\pi^{21} + \frac{1}{12}\tilde{H}_\pi^{21} - \frac{\sqrt{5}}{12}\tilde{H}_\pi^{21}, \\
B_{12} &= \frac{\sqrt{6}}{6}H_\pi^{21} + \frac{\sqrt{30}}{18}H_\pi^{21} - \frac{\sqrt{3}}{12}\tilde{H}_\pi^{21} - \frac{\sqrt{15}}{36}\tilde{H}_\pi^{21},
\end{aligned}$$

$$\begin{aligned}
B_{13} &= \frac{\sqrt{10}}{12}H_\pi^{21} + \frac{\sqrt{2}}{12}H_\pi^{21} - \frac{\sqrt{5}}{12}\tilde{H}_\pi^{21} + \frac{1}{12}\tilde{H}_\pi^{21}, \\
B_{14} &= -\frac{\sqrt{30}}{12}H_\pi^{21} + \frac{\sqrt{6}}{36}H_\pi^{21} + \frac{\sqrt{15}}{12}\tilde{H}_\pi^{21} + \frac{\sqrt{3}}{36}\tilde{H}_\pi^{21}, \\
B_{15} &= \frac{\sqrt{5}}{12}H_\pi^{21} - \frac{1}{12}H_\pi^{21} - \frac{\sqrt{10}}{12}\tilde{H}_\pi^{21} + \frac{\sqrt{2}}{12}\tilde{H}_\pi^{21}, \\
B_{16} &= -\frac{\sqrt{15}}{12}H_\pi^{21} - \frac{\sqrt{3}}{36}H_\pi^{21} + \frac{\sqrt{30}}{12}\tilde{H}_\pi^{21} + \frac{\sqrt{6}}{36}\tilde{H}_\pi^{21},
\end{aligned}$$

$$\begin{aligned}
B_{17} &= -\frac{\sqrt{2}}{4}H_{\pi}^{21}, \\
B_{18} &= \frac{\sqrt{6}}{4}H_{\pi}^{21}, \\
B_{19} &= -\frac{\sqrt{3}}{6}H_{\pi}^{21} - \frac{\sqrt{6}}{12}\tilde{H}_{\pi}^{21}, \\
B_{20} &= \frac{1}{2}H_{\pi}^{21} + \frac{\sqrt{2}}{4}\tilde{H}_{\pi}^{21}, \\
B_{21} &= -\frac{\sqrt{6}}{12}H_{\pi}^{21} + \frac{\sqrt{3}}{6}\tilde{H}_{\pi}^{21}, \\
B_{22} &= \frac{\sqrt{2}}{4}H_{\pi}^{21} - \frac{1}{2}\tilde{H}_{\pi}^{21}, \\
B_{23} &= -\frac{\sqrt{2}}{4}\tilde{H}_{\pi}^{21}, \\
B_{24} &= \frac{\sqrt{6}}{4}\tilde{H}_{\pi}^{21},
\end{aligned}$$

#### Appendix D: Spin configurations of the open charm final states

When the tetraquarks decay into the open charm modes, the decay matrix elements contain many terms. In order to avoid the complicated and lengthy expressions, we only collect the spin configurations of various open charm final states below. One can first compare the spin configurations of the S-wave and P-wave hidden-charm tetraquark states in Appendix A and different open charm final states. If the initial and final states have one or more common spin configurations, such a strong mode is allowed under the heavy quark symmetry. Otherwise, such a decay is suppressed.

S-wave  $D^{(*)}\bar{D}^{(*)}$

$$\begin{aligned}
B\bar{B} (0^{++}) &= \frac{1}{2}(0_H \otimes 0_l)_{0^{++}}^{++} + \frac{\sqrt{3}}{2}(1_H \otimes 1_l)_{0^{++}}^{++}, \\
B^*\bar{B}^* (0^{++}) &= \frac{\sqrt{3}}{2}(0_H \otimes 0_l)_{0^{++}}^{++} - \frac{1}{2}(1_H \otimes 1_l)_{0^{++}}^{++}, \\
B\bar{B}^* (1^{+-}) &= \frac{\sqrt{2}}{2}(0_H \otimes 1_l)_{1^{+-}}^{+-} - \frac{\sqrt{2}}{2}(1_H \otimes 0_l)_{1^{+-}}^{+-}, \\
B^*\bar{B}^* (1^{+-}) &= \frac{\sqrt{2}}{2}(0_H \otimes 1_l)_{1^{+-}}^{+-} + \frac{\sqrt{2}}{2}(1_H \otimes 0_l)_{1^{+-}}^{+-}, \\
B\bar{B}^* (1^{++}) &= (1_H \otimes 1_l)_{1^{++}}^{++}, \\
B^*\bar{B}^* (2^{++}) &= (1_H \otimes 1_l)_{2^{++}}^{++},
\end{aligned}$$

P-wave  $D^{(*)}\bar{D}^{(*)}$

$$\begin{aligned}
D\bar{D} \{^1P_1\} &= \frac{1}{2}(0_H \otimes 1_l)_{1^{--}}^{--} + \frac{\sqrt{3}}{6}(1_H \otimes 0_l)_{1^{--}}^{--} - \frac{1}{2}(1_H \otimes 1_l)_{1^{--}}^{--} + \frac{\sqrt{15}}{6}(1_H \otimes 2_l)_{1^{--}}^{--}, \\
D^*\bar{D}^* \{^1P_1\} &= \frac{\sqrt{3}}{2}(0_H \otimes 1_l)_{1^{--}}^{--} - \frac{1}{6}(1_H \otimes 0_l)_{1^{--}}^{--} + \frac{\sqrt{3}}{6}(1_H \otimes 1_l)_{1^{--}}^{--} - \frac{\sqrt{5}}{6}(1_H \otimes 2_l)_{1^{--}}^{--}, \\
D\bar{D}^* \{^3P_1\} &= \frac{\sqrt{2}}{2}(0_H \otimes 1_l)_{1^{--}}^{--} - \frac{\sqrt{2}}{2}(1_H \otimes 1_l)_{1^{--}}^{--}, \\
D^*\bar{D}^* \{^5P_1\} &= \frac{1}{3}(1_H \otimes 0_l)_{1^{--}}^{--} - \frac{\sqrt{3}}{3}(1_H \otimes 1_l)_{1^{--}}^{--} + \frac{\sqrt{5}}{3}(1_H \otimes 2_l)_{1^{--}}^{--},
\end{aligned}$$

where the label  $^1P_1$  in the above denote that the spins of  $D$  and  $\bar{D}$  are coupled into the total spin 0, then the total spin 0 is coupled with the P-wave orbital angular momentum into the total angular momentum 1. The labels  $^3P_1$  and  $^5P_1$  have the similar meanings with  $^1P_1$ .

S-wave  $D_{0,1,2}^{(*)}\bar{D}^{(*)}$

$$\begin{aligned}
 D_0\bar{D}^* + c.c. &= \frac{1}{\sqrt{2}}[(0_H \otimes 1_l)_1^{--} + (1_H \otimes 1_l)_1^{--}], \\
 D'_1\bar{D}^* + c.c. &= (0_H \otimes 1_l)_1^{--}, \\
 D_1\bar{D}^* + c.c. &= \frac{1}{\sqrt{10}}[-(0_H \otimes 1_l)_1^{--} + 3(1_H \otimes 1_l)_1^{--}], \\
 D_2\bar{D}^* + c.c. &= -\frac{1}{\sqrt{2}}[(0_H \otimes 1_l)_1^{--} + (1_H \otimes 1_l)_1^{--}], \\
 D'_1\bar{D} + c.c. &= \frac{1}{\sqrt{2}}[-(0_H \otimes 1_l)_1^{--} + (1_H \otimes 1_l)_1^{--}], \\
 D_1\bar{D} + c.c. &= \frac{1}{\sqrt{2}}[-(0_H \otimes 1_l)_1^{--} + (1_H \otimes 1_l)_1^{--}].
 \end{aligned}$$